FUN*da*MENTALS of Design

Topic 5

Power Transmission Elements I

Power Transmission Elements I

There are many ways to store or generate power, but all are useless without a *transmission* to transmit the power from a source to the place where it is needed in a form in which it can be used. Furthermore, the basic physics of power generation and transmission indicate that high speed and low torque are more efficient to create than low speed and high torque. The former is generated by high voltage and low current which is more economical than low voltage and high current. Once again, a transmission is often required to transform power into a useful form. But what combination of elements provides the best price/performance? For example, how should high speed low torque rotary motion from a motor be transformed into low speed, high force linear motion?

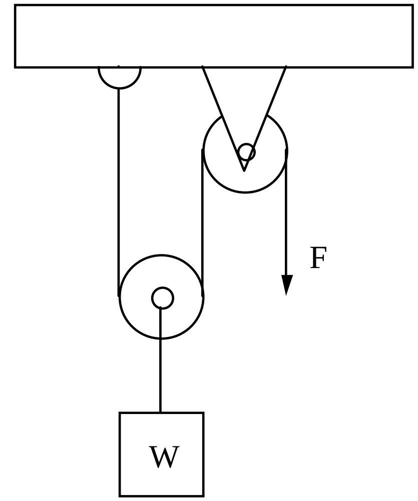
By designing machines in a modular fashion, an engineer can assume power will be provided in a certain manner. The engineer can then proceed with the design of the modules and elements that will transmit and utilize the power. This also allows some flexibility in optimizing the power source as the design precedes. For example, initial calculations may have indicated that a certain size motor was required, but in designing the power transmission system, the motor size may decrease/increase depending on the inertia and efficiency of the power transmission system. Accordingly, this chapter will focus on some of the elements of transmission systems, such as pulleys and wheels, belts and couplings. Chapter 6 will discuss screws and gears. Chapter 7 will discuss selection of optimal transmission ratios and actuators that use the elements in Chapters 5 and 6.

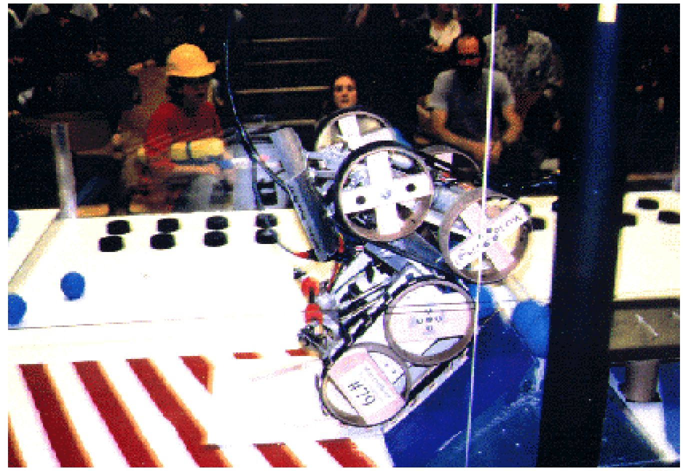
There are an amazing array of transmission components, and engineers need to become familiar with what is available, and what niche each element occupies. By applying FUNdaMENTAL principles and Maudslay's Maxims, you will better be able to separate marketing hype from engineering reality when you consider:

- Kinematics: motion, accuracy, space
- Dynamics: forces, speeds, life
- Economics: design, build, maintain

By creating machines for robot design competitions, you will have to use fundemental principles to help create the transmissions you need from very basic elements which will greatly hone your design skills. Remember to look at the world around you from garden tractors to hot-rods. Keep your eyes open!

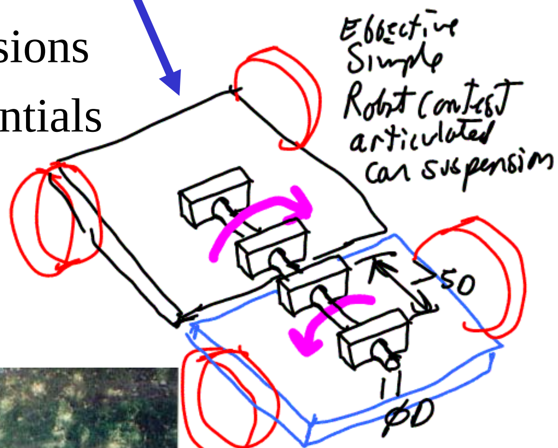
Topic 5
Power Transmission Elements I

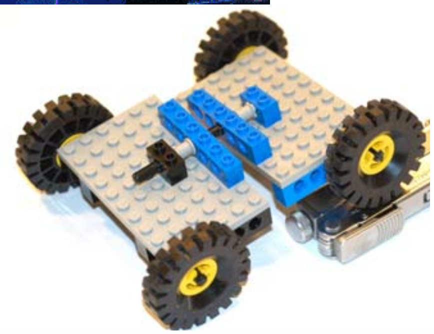


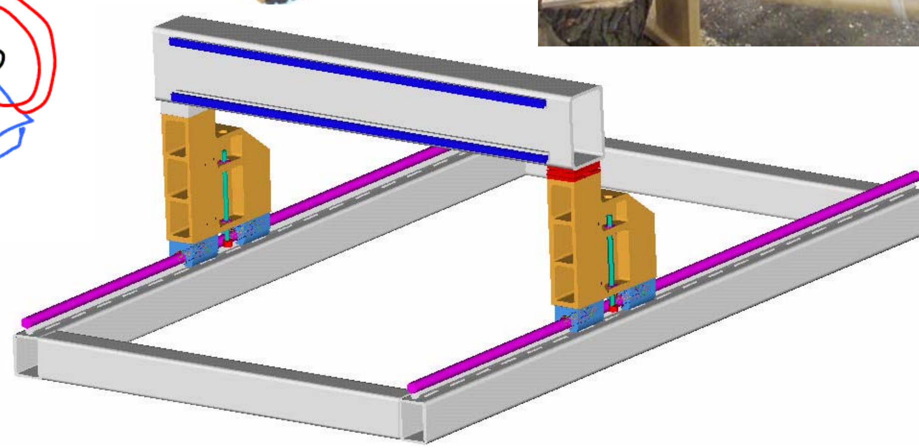


Topics:

- Transmissions
- Pulleys
- Winches
- Belts & Cables
- Wheels
- Steering & Suspensions
- Clutches & Differentials
- Cams
- Shafts
- Couplings







Pulleys

Pulleys are one of the oldest and most ubiquitous power transmission elements, but they require careful design. If a belt or cable runs around a fixed shaft, friction between the belt and the shaft can cause the efficiency to be low, and the cable can rapidly wear. A pulley reduces these effects with rolling contact between the cable and the machine, but it must be of sufficient size, typically 20 times the cable diameter, to prevent fatiguing of the cable strands. In addition, a crown on a pulley can be used to keep a flat belt centered on the pulley even if the pulley is not perfectly aligned (see page 5-10).

Pulleys transmit power from one location to another, and they can form a transmission ratio. The pulley equivalent of a lever and fulcrum is in fact *Atwood's machine*, which is comprised of a pulley mounted to a fixed object, and a pulley that moves with the cable and to which a lifting hook is attached. Since the tension in the cable is everywhere equal, one force input to the cable becomes two forces acting on the output pulley, and the force is doubled, although the lifting speed is half that of the cable. For multiple strand sheave system (like seen on large cranes), $Work_{in} = efficiency*Work_{out}$:

$$F_{out} = \eta F_{in} N_{number\_of\_cable\_strands}$$

$$V_{out} = \frac{V_{in}}{N_{number\_of\_cable\_strands}}$$

Large cranes have many strands of cable running over many sheaves (pulleys) in order to keep the cable tension low, which enables a smaller diameter cable and lower torque winch to be used. On such systems, are the pulleys supported by rolling element bearings to increase system efficiency? If the belt or cable rides on a pulley of diameter *D* with inside sliding-contact-bearing-on-shaft diameter *d*, and coefficient of friction μ, the efficiency η is found from:

$$work_{out} = work_{in} - work_{friction\_losses}$$

$$\left(F_{out} D/2\right)\theta = \left(F_{in} D/2 - \left(F_{out} + F_{in}\right)\mu\, d/2\right)\theta$$

$$\eta = \frac{F_{out}}{F_{in}} = \left(\frac{D - d\mu}{D + d\mu}\right)$$

As the plot shows, if the coefficient of friction can be kept below 0.1, very doable for a properly lubricated plain bearing, and the ratio of the pulley to shaft diameter is above 4, then efficiencies of greater than 90% can be easily obtained. Consider a system with D/d = 8 and 10 sheaves in the system, so the system efficiency would be 0.9753^{10} or about 78%. If the sheaves are supported by needle roller bearings, the coefficient of friction can be as low as 0.005, and the efficiency of 10 sheaves would be 0.9888^{10} or about 98.76%. *Which system should be used, a simple plain bearing system, or a more expensive needle roller bearing system? It depends on the functional requirements of the system, including cost and maintenance requirements! The analysis thus becomes your guide to selecting the proper machine elements that are to be used!*

Although the plot shows that it is desirable to maximize the ratio of pulley to shaft diameter, the shaft must be sized to meet several functional requirements. The shaft must:

- Withstand applied loads without yielding: Recall that for a simply supported pin of length L, the maximum moment is FL/4
- Not deflect too much: If the shaft is cantilevered, the angular deflection can cause the cable to rub on the side of the pulley and cause premature failure. In severe cases, it can cause the belt to track off the pulley despite the crown. Simply supported shafts have zero slope in the middle and thus help maintain tracking.
- The shaft diameter must support a sliding bearing whose maximum pressure load and maximum PV ratings[1] are not exceeded.
  - The bore and the faces of a plastic pulley typically act as their own radial and thrust sliding contact bearings. A metal pulley should have modular sliding contact bearing elements for radial and thrust loads.

Look at the mechanisms under the glass of a photocopy machine, and observe its cable drive. How does it use pulleys? Look closely at a crane as it operates. Look at the belts on the engine in a car. How might your machine use pulleys and how can you mount them so that they do not tilt under load?

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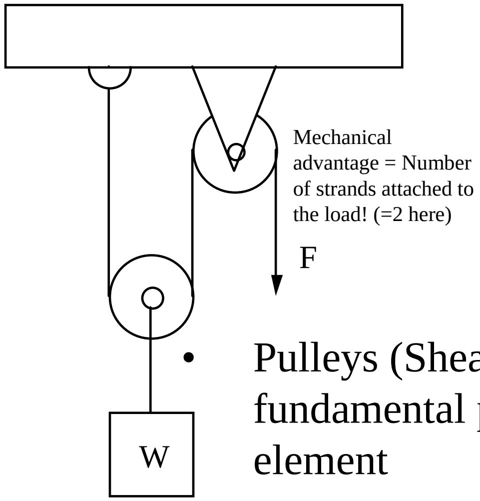
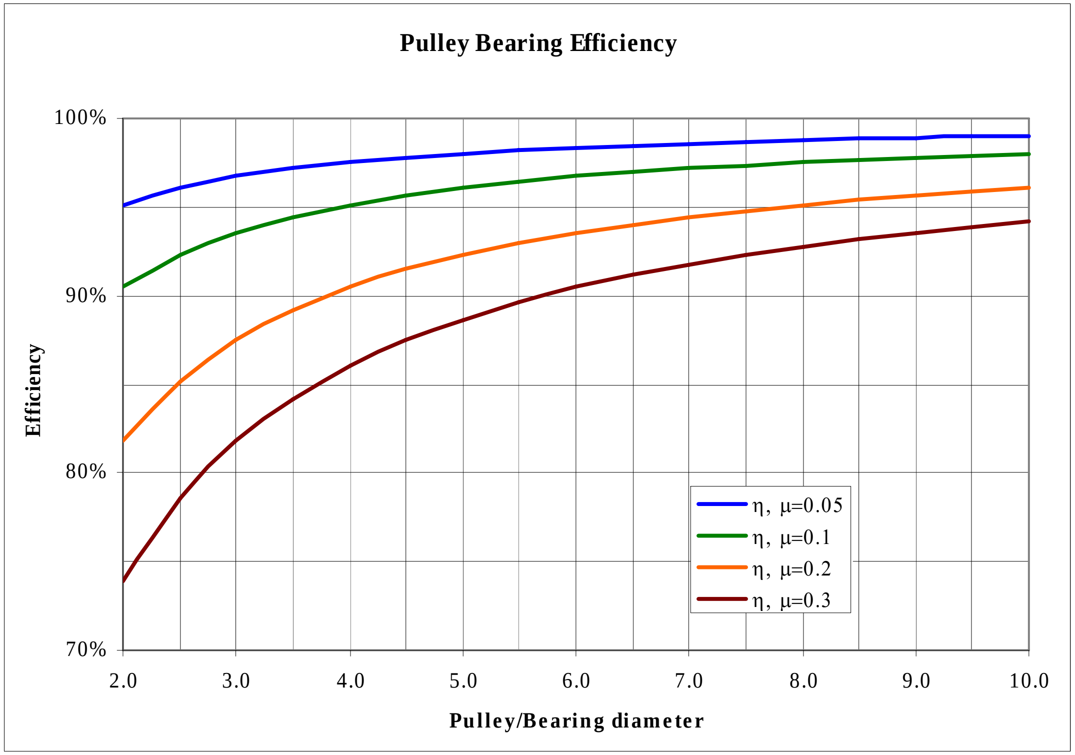
1. PV = (pressure)(velocity), and is a critical parameter for sliding contact bearings, as discussed in Chapter 10.

Pulleys



- Pulleys (Sheaves) are a most fundamental power transmission element
  - Mechanical advantage
  - Capstans
  - Efficiency
  - Tracking
  - Mounting



Pulleys: *Capstans*

Before there were pulleys that could rotate on a shaft, a rope was likely laid over a round bar in order to allow a person to pull down and have a load be hoisted. Someone probably figured out that they were having to pull down with more force than the weight of the load. They probably then made the connection with the wheel and thus the pulley was born; however, reciprocity would indicate that there could be some use for this effect, and as every sailer will tell you, a *capstan* is an indispensable element. A capstan functions as sort of a one-way brake. It allows you to pull in a rope, yet with mild effort, you can resist very large forces that are attempting to pull the rope back out.

The derivation of the capstan equation is very straightforward and also is used to obtain the breaking torque that can be obtained when a capstan is applied as a *band break*. A band break uses a band anchored at one end which passes over a rotating drum and is pulled at the other end to brake the drum. When configured as shown with the direction of rotation pulling against the band segment that is anchored, the principle of self help also applies. As the free-end of the band is pulled, the friction between the band and the shaft pull harder on the fixed end. The limit of the tension in the fixed end is a function of the wrap angle and the breaking force F that is applied. For a capstan and a brake, the cable tension T_2 that can be held by applying a tension T_1 for a wrap angle θ and a coefficient of friction μ is found by considering a differential element. The output tension is equal to the input tension plus the product of the coefficient of friction with the normal force caused by the two tensions. This normal force is:

$$N = (T + dT)\sin\left(\frac{d\theta}{2}\right) + T\sin\left(\frac{d\theta}{2}\right) \approx T d\theta$$

$$T + dt = T + \mu T d\theta \qquad \int_{T_1}^{T_2} \frac{dt}{T} = \int_0^{\theta} \mu d\theta \qquad T_2 = T_1 e^{\mu\theta}$$

$$\Gamma_{brake} = \mu\left(\frac{D_{brake}}{2}\right)\left(\frac{T_1 + T_2}{2}\right) \Rightarrow \frac{T_2\left(1 + e^{-\mu\theta}\right) D_{brake}}{4}$$

The brake torque is a function of the maximum normal force, which is the vector sum of the tensions T_1 and T_2, the brake drum radius $D_{brake}/2$, and the coefficient of friction, which can be easily found using the law of cosines. Is there thus a maximum brake torque angle and how can one find it?

$$N_{brake\_max} = \sqrt{T_1^2 + T_2^2 - 2T_1T_2\cos\theta}$$

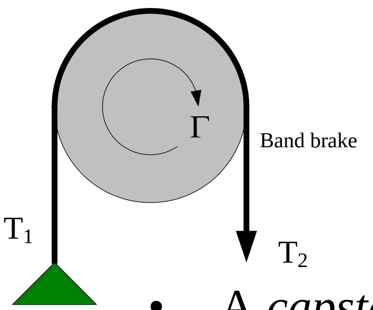
$$\Gamma_{brake} = \frac{\mu N_{brake\_max} D_{brake}}{2}$$

$$2\mu e^{\mu\theta} - \mu\cos\theta + \sin\theta\Big|_{=0\_for\_optmal\_wrap\_angle}$$

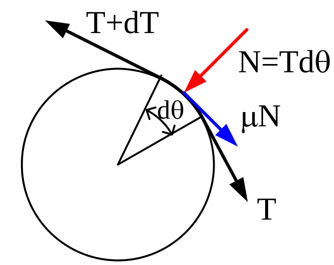
Enter numbers in **BOLD**, **results in RED**			
Capstan		**Band brake**	
Angle of wrap, theta (degrees)	**180**	Brake diameter, Dbrake (m)	**0.1**
Coefficient of friction, mu	**0.10**	Brake torque factor, BTF	**2.37**
Capstan ratio, CR	**1.37**	Maximum brake torque (N-m)	**0.1185**
Holding force, Fhold (N) (Fbrake)	**10**	**Optimal wrap angle (use Goal Seek)**	
Resistable pulling force, Fpull (N)	**13.69**	Optimal angle, phi (degrees)	**202**
Cable pulling around a shaft		d/dθ =0	**3E-03**
Net force out	**7.30**	Capstan ratio, CRO	**1.42**
Efficiency	**73%**	Brake torque factor, BTFO	**2.38**
		"Optimal" brake torque (N-m)	**0.1190**

In terms of the analysis process, note that once the equation for the tensions in the band is found, the next step was to create a hypothesis to find the normal force on the shaft. In the case of a wrap angle of 180 degrees, the sum of the two forces is the force the shaft must resist and so vector addition of the tensile forces makes sense. If the wrap angle was 360 degrees, the net force on the shaft would be less than the maximum tension force. On the other hand, the capstan force may be much larger, so a conditional “MAX” statement can be used in the spreadsheet to use the greater of the tension forces in the band, or the vector sum of the forces. These relations can be used to design brakes or to determine the efficiency of putting a cable over a round shaft to see if it is worth it or not to make a pulley. In industrial practice one would likely never do the latter, but in a robot contest for a quick small trigger mechanism, for example, it might be acceptable.

How can you use the mechanical advantage provided by a capstan, or how can you tolerate the losses incurred with just running a cable over a round shaft? Reciprocity strikes again!



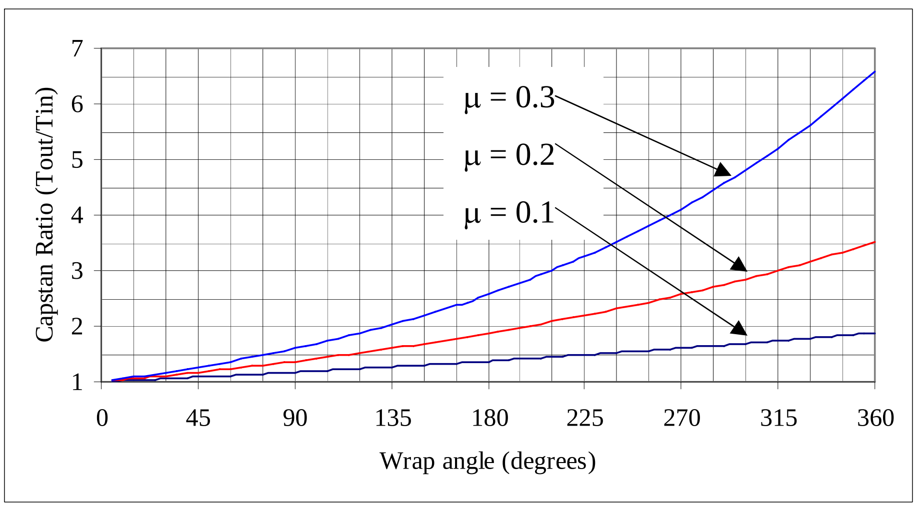
Pulleys: *Capstans*



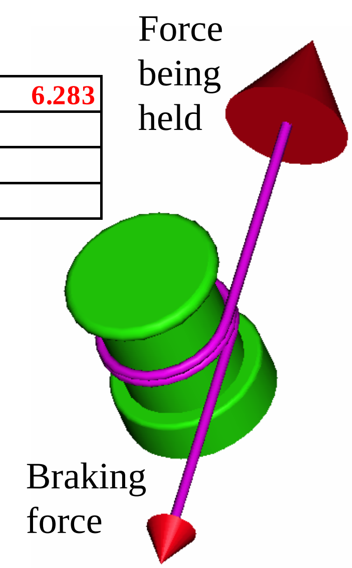
- A *capstan* is typically a fixed, or controlled rotation (with one-way clutch), body-of-revolution which a cable wraps around
  - A capstan can also form the basis for a band brake, where a band is anchored to a structure, and then wraps around a shaft…
- A cable wrapped around a capstan by θ radians with coefficient of friction m and being held with a force F_{hold}, can resist the pull of a cable with many times higher force F_{pull}

$$F_{hold} = F_{pull} e^{-\mu\theta}$$

- If a belt or cable runs around a fixed shaft, then there is a lot of friction between the belt and the shaft, and the efficiency is low:



Angle of wrap (degrees, radians) θ	**720**	**6.283**
Coefficient of friction μ	**0.2**	
Holding force Fhold	**1**	
Pulling force that can be resisted Fpull	**3.52**	



Winches

A *winch* is a device used to control the tension and position of a *cable* by winding it onto a *drum*. Winches are one of the principle elements of cranes, as they provide the lifting force for the hook and often the boom. Winches are also commonly used to provide a pulling force for equipment such as tow trucks. Until the introduction of hydraulics, winches were also one of the principle means of actuation for construction equipment. Robots, particularly those used in competitions, may use a winch to help pull themselves up a steep slope, or to deploy a wall. In accordance with the FRDPARC design tables, consider first the primary function requirements for a typical winch:

Table 1:

Functional Requirements	Design Parameters	Analysis
High cable force & speed	Motor/gearbox torque/speed Drum diameter	Small diameter needed to minimize motor torque, or use large gear ratio
Large cable capacity	Drum diameter, length	Beware of increasing effective diameter on cable force, see spreadsheets
Even cable winding	Drum length Fairleader	Difficult to analyze, run experiments or seek similar solutions

Given these primary issues, consider the remaining corresponding issues that are most often overlooked by designers, but which are critical to the design process:

Table 2:

References	Risks	Countermeasures
Motor gearbox catalogs, Commercial winch companies (search the internet)	Static holding force could cause electric motor to burn out	Use a brake or non-back drivable transmission (worm gear). Cranes often use hydraulics.
Commercial winch companies	Short length, large diameter can make the effective cable diameter increase creating need for large motor.	Bearings and structure are cheaper than motors, use a longer drum
Commercial winch companies	Uneven cable winding can reduce capacity	Good cable management, do not skimp on the fairleader

The first step in winch design is to evaluate the amount of cable and force needed and then to determine the drum size and motor/gearbox that is required. For a simple winch to evaluate the cable force and drum wraps, the spreadsheet *WinchCable.xls* can be used:

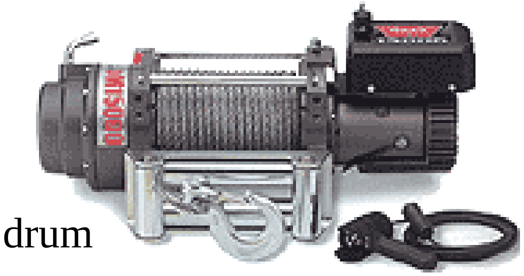
Enters numbers in **BOLD**, Results in RED			
Motor Torque (N-m)	**0.4**		
Drum Diameter (mm)	**40**		
Drum Length (mm)	**30**		
cable diameter (mm)	**1.5**		
wrap number	mm of cable/wrap	Total length	Maximum cable force
1	2513	2513	20
2	2702	5215	19
3	2890	8105	17
4	3079	11184	16

For the case of a winch raising a boom, the spreadsheet *Winchboom.xls* can be used to select the detail design parameters. In terms of detailed design considerations, note the picture that shows what looks like a good winch, except that the drum was connected directly to the motor. *Even for a robot design competition, haste makes waste.* Typically the drum must be supported by its own bearings because it is long with respect to the motor shaft diameter and the motor bearings would be overloaded by a large radial load far from the motor's front bearing. Torque from the motor should be applied via a flexible coupling, universal joint, or a well-aligned spline, else misalignment from manufacture or deflection of components from the applied load could deform the motor shaft and cause premature wear of the bearings.

A *fairleader* is a flared opening that guides the cable in and helps it wind properly. The simplest form is just a smooth rounded opening which ideally the cable never even touches. When the cable pulls an object from the side of the winch, a fairleader with rollers may be required as can be determined by the capstan equations. A powered fairleader moves back and forth ensuring that the cable winds in an orderly fashion without randomly overlapping itself.

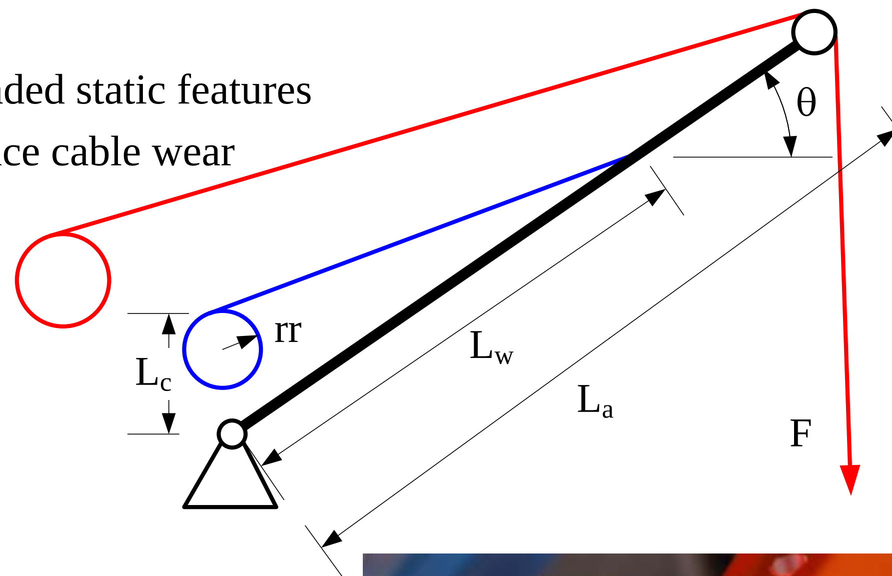
How would you optimize motor/gearbox size with drum size to pack your winch in as small a space as possible? Do you have enough torque so a large diameter short length drum could be attached directly to the gearbox output shaft, or do you need a long drum supported at each end by bearings?

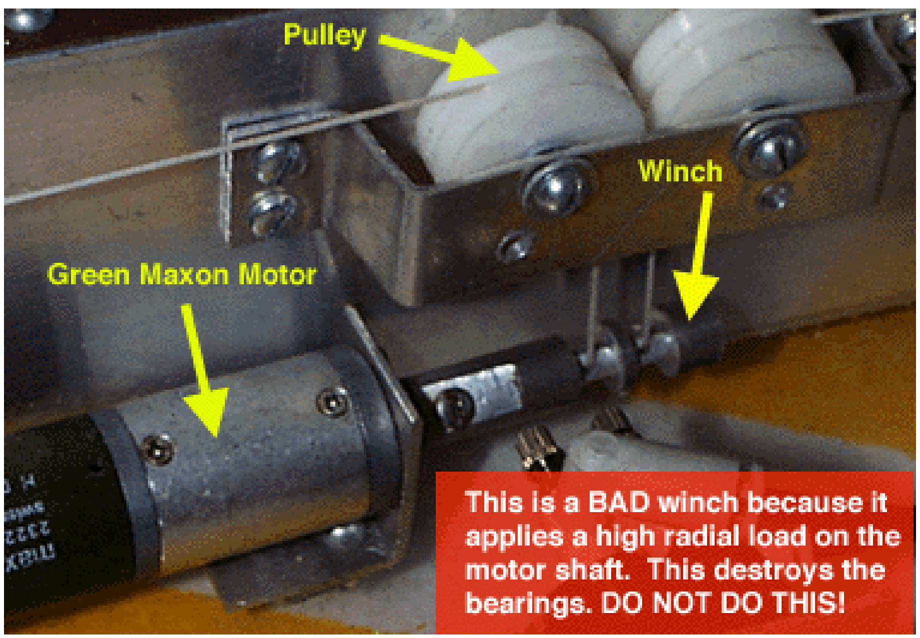
Winches

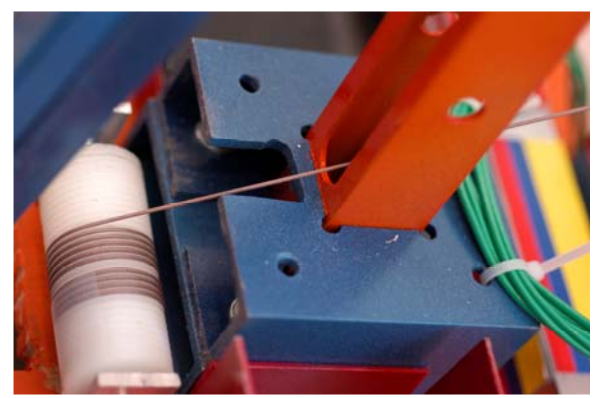


http://www.gowarn.com/winches.htm

- A winch is a motorized drum that controls cable tension
  - A single wrap of cable on the drum requires a long or large diameter drum
    - Effective drum diameter and winch force capability remain constant
  - Multiple wraps of cable on the drum allow for more cable in a smaller place
    - Drum diameter and winch force capability vary
  - A fairleader is a device to control the input/output of the cable so it winds on the drum in an orderly fashion
    - The simplest design just uses smooth rounded static features
    - Vertical and horizontal roller designs reduce cable wear







Belts & Cables

Belts and *cables* are very common power transmission elements because their elastic nature enables them to pass over round objects (pulleys) typically with a high degree of efficiency. The term *power transmission* literally means the power output from the device equals the product of the efficiency and the power input to the device. Therefore, analyzing any power transmission system really is as simple as keeping track of the products of speeds and torques (or forces) and efficiencies.

Belts and cables are laterally compliant, which makes them forgiving of misalignment, so they are commonly used applications from automobiles to office equipment. In all cases, the key functional requirements of a belt or cable are to transfer tensile loads and to pass over a pulley. The life of a belt or cable is a function of its pre-tension, the diameter of the smallest pulley, and the load it is expected to carry. The key efficiency issue is to minimize contact at different diameters to prevent differential slip between the belt and the pulley. The belt cross section must be chosen to withstand the sum of the stresses from these three sources while maintaining a desired level of stiffness.

A *cable* is a flexible tensile element whose elements are stranded together such that all of the strands share the load. When the cable passes over a pulley, the strands locally slip over each other so the bending stress in any single strand is far lower than if a solid bar of the same diameter were bent over the pulley. However, slip dissipates energy, so cable drives are used primarily in low speed applications. The load that cables can carry is a function of the pre-tension, the coefficient of friction between the cable and the pulley, and the wrap angle. Hence the pulley acts like a rotating capstan.

A *belt* can have many forms, but the three principle types are: *flat*, *toothed*, and *vee* belts. Flat belts transfer a load by being pre-tensioned, and as with cables, the pulleys act like capstans. Flat belts are generally very efficient, unless forced to run on crowned pulleys to accommodate misalignments (see page 5-10). A flat-toothed belt positively engages the pulley, and requires only enough tension such that the load acting on the tooth angle does not create a force large enough to pop the tooth out of the pulley. The efficiency is still high because the tooth shape on the belt and the pulley are designed to minimize differential slip[1]. A Vee belt uses the principle of self-help, whereby the greater the load, the more the Vee shape wedges itself into the pulley. This increases the torque transmission capacity but decreases the efficiency.

The lateral flexibility of belts and cables can accommodate errors in rigid elements. The *kinematics* of belt and cable drives allow them to achieve the desired motion with a great deal of accuracy in a modest space. In particular, their lateral compliance allows system components to be mounted with greater ease than gears or screws. For example, cables can be routed around pulleys whose axes of rotation are not necessarily parallel to each other. With flat belts, one must use crowned surfaces or edge guides to deal with the invariable tendency for a belt to "walk off" a pulley (see page 5-10).

The *dynamics* of belts and cables, which often contain reinforcing elements such as steel or Kevlar™, make them efficient power transmission elements. Not only can high power often be transmitted, but high force or high torque can also often be transmitted very repeatably. On the other hand, it is difficult to obtain the stiffness or load capacity of gears or screws. Speeds are limited by unsupported lengths of belts or cables vibrating. Fortunately, measuring the frequency of vibration is straightforward and it can be used as an accurate measure of belt tension. Typical linear systems may achieve speeds of many meters per second, and rotary systems may run at thousands of rpm.

The *economics* of belt and cable system design are very good precisely because of their ability to elastically average errors, and similarly, they can be very straightforward to build and maintain. Key factors affecting cost include not only the belt and pulleys, but also managing belt or cable stress around pulleys, maintaining proper belt or cable tension, and achieving reasonable alignment of the pulleys.

For every linear and rotary motion you are contemplating in your robot, sketch how you could use a belt or cable as the power transmission element. What are the limits on pulley size and location? What sort of speeds and loads must be accommodated? Can a belt or cable enable more convenient placement of a motor? How might you control belt or cable tension? How might you change the belt or cable if it were to become damaged?

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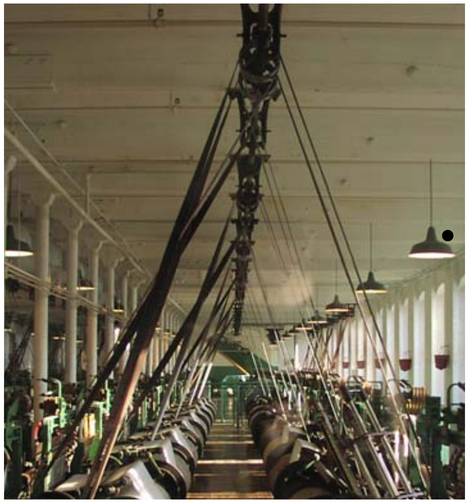
1. Chains operate on a similar kinematic principle, and the use of high strength steel enable them to carry much higher loads; however, chains require lubrication, but other than that, can be designed into systems with essentially the same methods.



Thanks to Prof. John Lienhard for this photo!

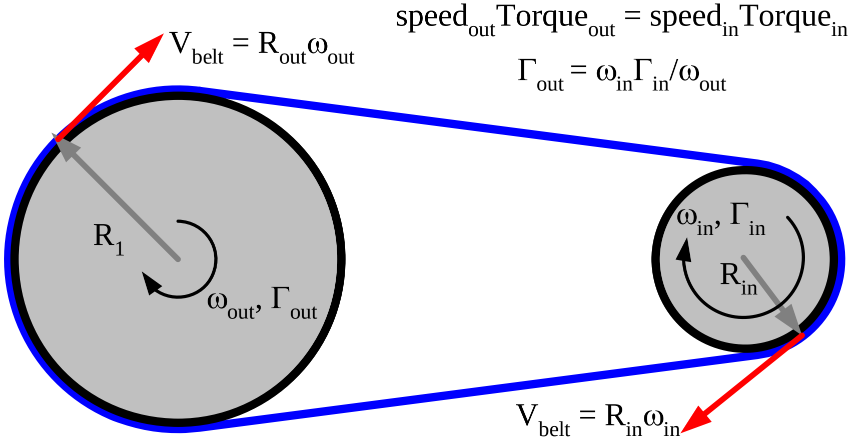
Belts & Cables

- Applications and Engineering
  - Linear motion
  - Rotary motion
  - Crawler tracks
  - Manufacturing & Assembly

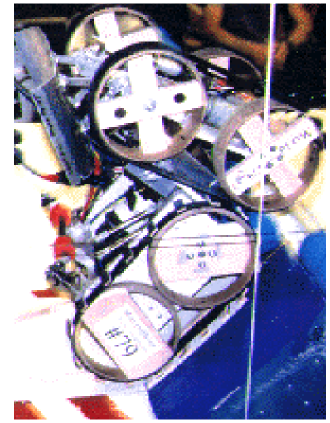
$$Power_{out} = Power_{in}$$

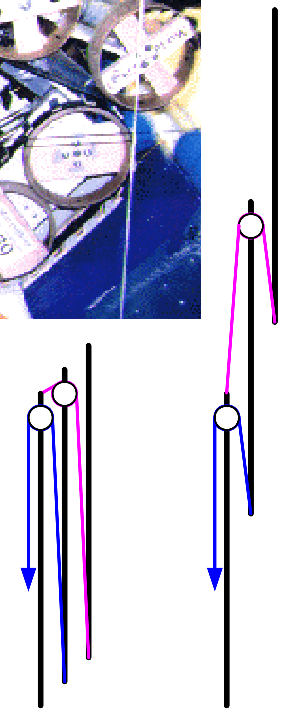
$$speed_{out}Torque_{out} = speed_{in}Torque_{in}$$

$$\Gamma_{out} = \omega_{in}\Gamma_{in}/\omega_{out}$$



Bishop Brady High School FIRST robotics team 2005 robot (Prof. Slocum was the coach)





http://www.genielift.com/ml-series/ml-1-5.html

Belts & Cables: *Stress, Tension, & Center Distance*

To determine the belt cross section required to carry a load *F*, pretension *T* and stresses induced by passing around the pulley of diameter *D*, start with determining the diameters of the pulleys needed. The pulleys should meet the optimum transmission ratio requirements, and be of a minimum size for the cable diameter or belt thickness. The minimum pulley size depends on the type of cable or belt used, and is best determined using design charts provided by the manufacturer, or from a design handbook. In general, the stress is inversely proportional to the pulley diameter. A thin flat belt of modulus *E*, poisson ratio η, conforming to a pulley of diameter *D*, undergoes plain strain[1], and the resulting stresses in the belt are determined in *bandstress.xls*:

$$\sigma_{load} = \frac{F}{wt} \qquad \sigma_{pre-tension} = \frac{T}{wt} \qquad \sigma_{pulley} = \frac{tE}{D\left(1-\eta^2\right)}$$

A key factor in manufacturing systems with belts or cables is the space constraint imposed by pulley size and center distance. As a primary factor, the need for a reasonable pulley diameter can have a huge effect on the space required for the design. For a flat metal belt, proper pulley size is based on the stress it produces in the belt. For a molded rubber (including reinforced) belt or cable, the ratio of pulley diameter to cable diameter depends on how the belt is reinforced or the number of strands in the cable. The pulley diameter should be 30 to 50 times the belt thickness or cable diameter. For string and hemp rope, 10:1 is the minimum ratio. The larger the ratio, the greater the life.

Once the pulley diameter is selected, the center distance can be determined based on the total amount of space available, the length of available belts, and the need for a tensioning system. The law of cosines provides the basis for finding belt length as a function of pulley diameter and center distance, but finding the angle at which the belt is tangent to the pulley results in non linear equations best solved iteratively. The spreadsheet *pulleycenterdistance.xls* can easily accomplish this calculation using the "Goal Seek" tool in Excel™ to find a center distance that drives the error value to zero. For example:

Enters numbers in **BOLD,** Results in **RED**	**Toothed**	**Flat**
Belt pitch, P	**4**	
Number of teeth, N	**50**	
Start with a guess for C, and then use Goal Seek		
Center distance, CD	**35.76**	
Large pulley pitch radius, R2	**25**	
Small pulley pitch radius, R1	**15**	
Length of belt, Lt, Lf	**200**	**200**
Tangent segment, A	**34.3340**	
gamma, g	**0.2834**	
Phi, f	**1.2874**	
Theta, t	**1.8542**	
error	**0.000**	**0.000**

With respect to tension in the belt or cable, the capstan effect can be used to determine the required tension needed to transmit a desired torque or force, such as done in the spreadsheet *capstan.xls*. How should the tension be determined? An estimate can be made by measuring the force *F* needed to deflect the belt sideways a small amount δ. A simple more accurate means is to pluck the belt and measure its frequency of vibration ω (Hz). Given its length *L* and mass per unit length *q*, the pre-tension *T* is determined in the spreadsheet *bandstress.xls* from (units of *N, m, kg, s* or *N, mm, g, s*):

$$T = \frac{FL}{2\delta} \qquad T_{Nmks} = (\omega L)^2 q \qquad T_{Nmmgs} = (\omega L)^2 q / 10^6$$

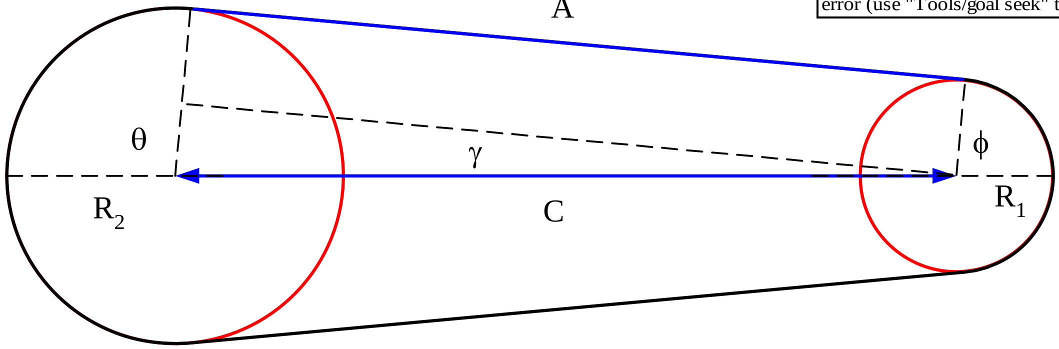
Consider the belts or cables you are planning on using, and determine the minimum pulley size to give you the life needed for your robot. In general, the pulley size shown in handbooks is for millions of cycles, so you may be able to use a smaller pulley (run an experiment!). What stresses and pre-tensions exist? Will you need to use a toothed belt? Will your belted module still meet the functional requirements of your concept?

1. The strain is all in a plane (the cross section remains rectangular)

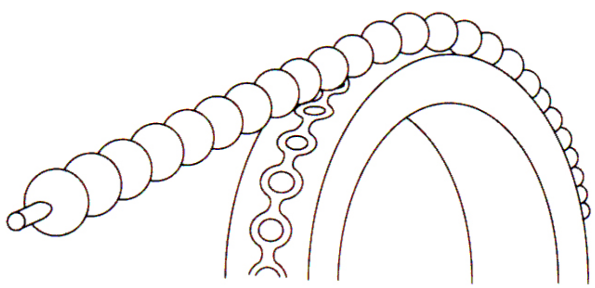
Belts & Cables: *Stress, Tension, & Center Distance*

- Belts and cables are very robust elements, but they require engineering of 3 basic details:
  - Stress from wrapping a belt or cable around a pulley
  - Tension in the belt or cable
  - Center distance between pulleys

pulley_center_distance.xls		
By Alex Slocum, last modified 2/19/04 by Alex Slocum		
Pulley Center Distance Calculation		
Enters numbers in **BOLD**, Results in **RED**		
	Toothed belt	
Belt pitch, P	**0.2**	
Number of teeth on belt N	**100**	Enter the number of teeth on the belt, if center distance ends up being too small, increase number of teeth
Start with a guess for C, and then use Goal Seek		
Desired center distance, C	8.24	
Large pulley pitch radius, R2	**0.5595**	
Small pulley pitch radius, R1	**0.5595**	
Length of belt, Lt, Lf	**20**	
Tangent segment, A	**8.2423**	
gamma, g	**0.0000**	
Phi, f	**1.5708**	
Theta, t	**1.5708**	
error (use "Tools/goal seek" to set value of B17 to zero by changing B9)	**0.000**	



Bandstress.xls	
By Alex Slocum	
Last modified 4/22/02by Alex Slocum	
Stress in a flat belt wrapped around a pulley	
Enters numbers in BOLD, Results in RED	
Belt parameters	
Thickness, t (mm)	**0.10**
Width, w (mm)	**5**
Modulus, E (N/mm^2)	**2.00E+05**
Poisson ratio, n	**0.29**
Forces	
Load to be carried, F (N)	**10**
Belt stress, sigF (N/mm^2)	**20.0**
Pulley wrap stresses	
Pulley diameter, D (mm)	**50**
stress	437
Motor torque required (N-mm)	**250**
Capstan effect	
coefficient of friction, mu	**0.2**
Wrap angle, q (degrees)	**180**
required pre-tenion, pT (N)	**5.3**
Belt stress, sigT (N/mm^2)	**10.6**
Total stress	**467**
Total strain	**0.23%**
Check: Tension in the belt (pluck it like a guitar string)	
Measured frequency of lateral vibration (hz)	**150**
Free-length (mm)	**300**
density (g/mm^3)	**7**
mass per unit length (g/mm)	**0.004**
Tension (N)	**7.1**



Bead belt drive

Belts & Cables: *Linear Motion*

There are many examples of belts and cables being used to convert motor torque and rotation to linear force and displacement, where generally linear motion bearings provide guidance for the load being moved. The belt or cable wraps around two pulleys, and the load is attached to the belt or cable in between the pulleys. Because the attachment zone is usually not intended to travel around the pulley, a spring or screw actuated tensioning mechanism can be used to maintain or create belt tension. When there is a need for the attachment zone to pass around the pulleys, special belts can be purchased, or a chain can be used. An example is the cable on a ski lift.

A load is often attached to the belt at a zone between two pulleys, and the belt or cable wraps half way around the drive pulley. As shown on page 5-7, this limits the capstan effect that can be achieved. For a given tension, if more force is required, multiple wraps around the drive pulley are needed. In this case, as the cable or belt winds in on one side of the drum and lets out from the other side, the helix angle formed by the cable diameter divided by the drum diameter implies that the cable will translate along the drum. Thus the drum needs to be long enough to give the cable enough room to translate. In addition, for precision motion and to prevent slip, there needs to be a portion of the cable that never leaves the drum. Otherwise there is slip caused by a portion of the stretched cable accumulating on the drum and then being let off the other side. Furthermore, tension needs to be maintained in the band. Maintaining tension is important if precision linear position is to be determined using a rotary encoder to measure drum revolutions. If unsure, run a bench-level experiment!

Because a cable's diameter is typically smaller than a belt's width, cables are often used for linear motion systems. In addition, the cable can be wrapped around a drive drum. This takes advantage of the capstan effect to minimize the required pre-tension and minimizes radial loads on the pulleys' bearings. A very common example of a linear motion system driven by a cable is the scanning shuttle in a photocopier. Open the lid and watch! If the total travel distance is L (+/- L/2), and the drum diameter is D, then with a safety factor of 1.2 to prevent slip, the number of wraps *N* on the drum should be:

$$N = 1.2\frac{L}{\pi D}$$

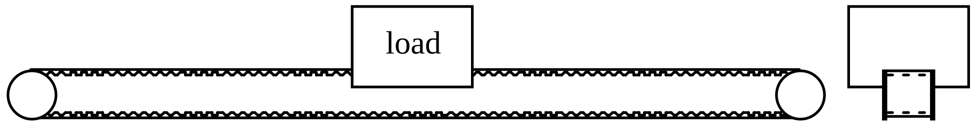
Consider a semiconductor test equipment application. The task was to index a tray of packaged semiconductor Devices Under Test (DUTs) under a test head and then plunge them into the test head's electrical contactors. As the picture shows, a steel belt can be used instead of a cable because a thin flat steel belt can conform to a smaller diameter pulley while having a much higher axial stiffness. The difficulty was that the devices had to be kept in an oven so they could be tested at 100 ^{o}C! It was desired to keep the servo motor and sensors outside of the oven. Note the two wraps of the steel belt around the capstan drive. Why is the *bench level prototype* system at an angle? The system is at an angle because of a software error that caused the axis to plunge before it was properly positioned. The design engineer working with the software engineer was assured that this could never ever happen; however, the mechanical engineer decided to mitigate possible software risk with the countermeasure of breakaway points!

As another example, consider the top view picture of the hockey puck dispensing module designed for the 2.007 robot competition *MechEverest*. The module moves a line of hockey pucks forward so they fall into a slot to score. Given space constraints, the force cannot be applied through the pucks' center of mass. The *Functional Requirements* of the module are to provide linear force and motion from a distance. This requires two design parameters: a mechanism to create the force, and a guidance mechanism to withstand the moment caused by the force. The force is created by a string that wraps around the capstan attached to the motor shaft and a pulley and is attached to a carriage. The carriage is made from a plastic block with two holes which rides on two welding rod shafts that provide guidance. Any load applied to the carriage creates a moment that is withstood by the welding rods, so no moment is applied to the cable.

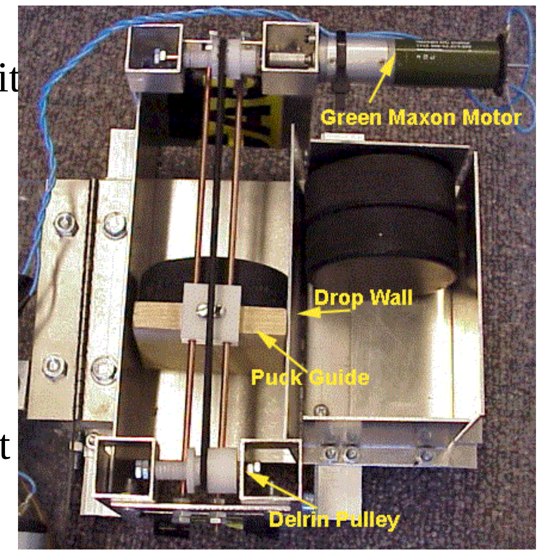
For each linear motion module for which you have created concepts, investigate the feasibility of using a belt or cable drive. Sketch concepts of the modules and use the spreadsheet *Bandstress.xls* to perform the design calculations. When is a capstan drive, where the cable makes several wraps around the drum attached to the motor, most effective, verses a simple belt that passes around two pulleys, one of which is driven?

Belts & Cables: *Linear Motion*

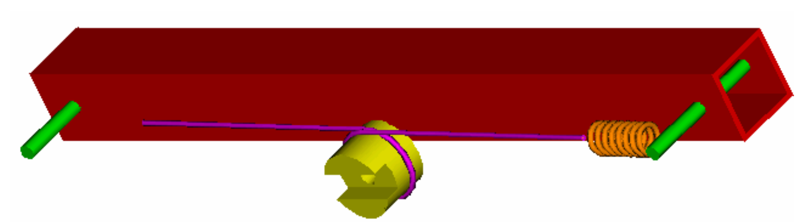
- Belts & Cables are a effective way to convert rotary to linear motion



- The force F in a belt with tension T on a pulley of diameter D that can be generated by the torque Γ can be conservatively estimated by:
    - $F = 2\Gamma/D$ *for toothed belts*, and $F = 2\Gamma/D$ for flat belts but is limit capstan effect
        - Play with *bandstress.xls*
  - The speed is simply
    - $V_{linear}=\omega_{motor}*D/2$
- Belts run on pulleys
  - For flat belts, pulleys must be crowned to accommodate misalignment
  - Timing belt pulleys must be aligned to prevent premature failure



Should this metal belt driven axis be at this angle? NO, it was the software (honest) that made the vertical axis plunge while the metal belt driven horizontal axis was still extended…



Belts & Cables: *Crawler Tracks*

The wheel is undoubtedly one of the greatest inventions in history. It enabled people to spread far and wide, and to travel and trade. An interesting historical artifact of wheeled vehicles are the tracks (ruts) they have left in many trails. In fact, there are places in the original Oregon trail where you can still see the ruts left by wheels. This also likely means that there was mud, and who has not been stuck in the mud or snow or tried to ride a bike in soft sand? With the advent of the steam engine came the tractor to replace the horse, and the weight of the vehicle and its payload increased dramatically; and with it, an even greater tendency to sink in the mud. At what point did someone think "there has to be a better way!"? It turns out that in the late 18th century someone thought to spread the load out onto a moving track. Was this inspired by a belt on a pulley? Later in the 19th century with the advent of steam power, who first thought that a steam powered locomotive could economically run on steel tracks that sat unused most of the time? We may never know.

As tractors grew larger and heavier, the size of wheels required to keep them from sinking became ridiculous. Then on November 24, 1904, Benjamin Holt replaced the rear drive wheels of a 40 horsepower Holt Junior Road Engine Number 77 with a pair of tracks nine feet long and 2 feet wide. The use of tracks not only helped keep tractors from sinking into the soft earth, reciprocally, they also helped keep the tractor from compacting the soft earth making fields easier to plow.[1]

Are crawler tracks merely used to keep tractors from sinking into the mud? Instinctively it seems that they should also have better traction; but why then do we even bother with wheels? Given the functional requirement to not slip (spin your wheels), there are two possible design parameters: 1) frictional contact between the surfaces, 2) interlocking contact between the surfaces.

If the strategy selected is to rely on friction contact, in general, the coefficient of friction, on a macro scale, is independent of the surface area of contact. The tractive effort is a product of the weight of the vehicle and the coefficient of friction between the wheels and the surface. Therefore, when creating detailed concepts for a frictional drive interface, selecting the best materials is the primary design parameter. In robot competitions, it is important to do simple tilting plane friction tests to determine which materials in your kit have the highest coefficients of friction with respect to the surfaces on which they will be driving. When doing these tests, you can experiment with the area of contact and the type of surface (smooth or carpeted) to see what is best. The secondary design parameter is the tread or surface features on the wheels. In real-world vehicles, the treads act to channel water out of the interface, so the rubber can make good contact with the road. The tread is also important when driving on softer surfaces, or on snow, where the tread helps to maintain traction by interlocking contact with the surface.

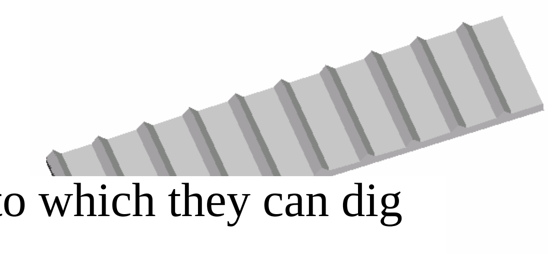
If the primary strategy is to utilize interlocking contact, then the principle design parameter is no longer the coefficient of friction, it is the shear strength of the contact region. In this case, the amount of tractive effort is a function of the area and geometry of the contact points and the shear strength of the materials in contact. Given the width w of each track, the number n of track elements on each track that are in contact with the ground, the length L of each track element (the pitch), and the shear strength τ of the ground (which is often locally higher because the weight of the tractor does some compaction!), the tractive effort of the machine will be $F = 2wL\tau$.

Tracked vehicles can use their incredible tractive effort to enable them to turn by driving one track forward while driving the other backwards so they can turn in place. This is called *skid steering*, but this results in large lateral forces on the tracks. Simple rubber belts on crowned rollers used as tracks on robots can come off when skid steering. Real tracked vehicles have a center guide feature that engages grooves in the guidance sprockets to resist lateral forces. Can you make center guiding ribbed features on a rubber belt?

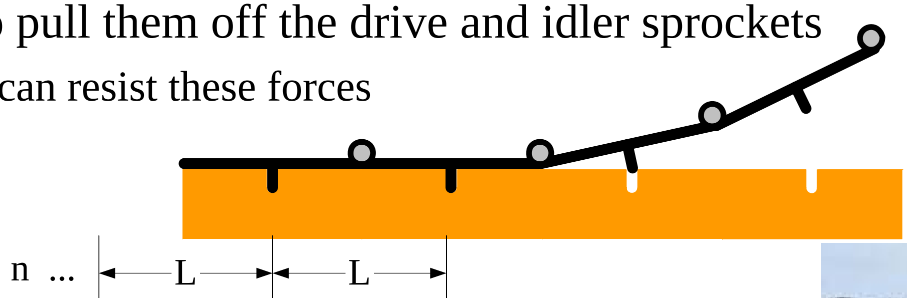
Note in the above, it is important to think of the process by which frictional versus interlocking contact means are considered. How then can you decide which system to use for a robot contest? What are the functional requirements for tractive force? If the competition surfaces are smooth, crawler tracks provide no direct tractive advantage. But carpet can be engaged by sandpaper belts... What are the functional requirements for mobility? Crawler tracks can also allow a robot to go over obstacles that may otherwise cause a wheeled vehicle to hang up in the middle. Do tracks need a suspension system like wheeled vehicles?

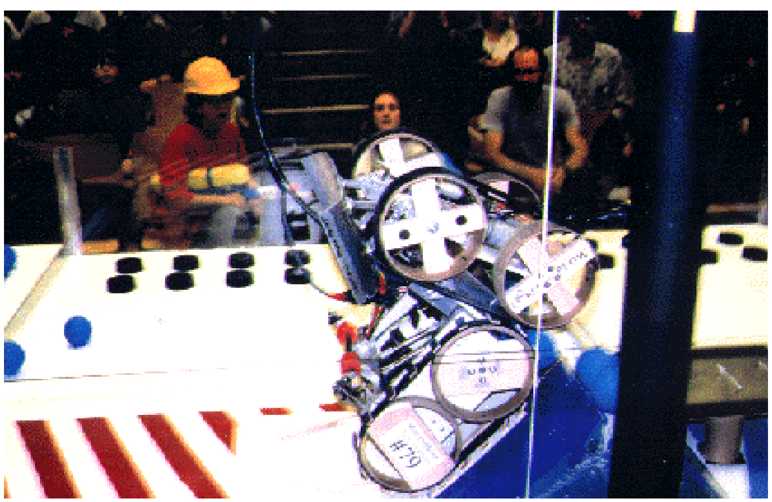
1. From www.caterpiller.com Also search on www.Google.com for *caterpillar history tractor*

Belts: *Crawler Tracks*



- Tracks primarily help when there is loose media or a surface into which they can dig
  - Track treads can be made by gluing rubber strips onto a belt's surface
- Tracks (and treads) usually do not help on smooth surfaces unless:
  - A "sticky" elastomer is used where apparent coefficient of friction also depends on contact area
    - Its apparent coefficient may actually be higher at lower contact pressure
  - Dust or dirt on the surface needs to be penetrated/pushed aside by tread teeth
- Tracked vehicles skid steer, and as a result there are often large lateral forces on the tracks which try to pull them off the drive and idler sprockets
  - A central ridge can resist these forces





www.caterpillar.com

Belts & Cables: *Rotary Motion*

Belts and cables were first created as rotary transmission elements. In the early years, a water wheel turned a large shaft that ran the length of a factory. Numerous pulleys located along the shaft transferred power from the main shaft to smaller shafts, which in turn often transferred the power to other machine elements. This system was used until electric motors were invented to deliver power where it was needed. Belts and cables are now used with electric motors to efficiently transfer power from one axis to another while also achieving modest transmission ratios. Belts and cables also do not require lubrication and they are generally tolerant of misalignment.

The *torque capacity* of a smooth belt transmission is largely affected by the capstan effect. This also effects the center distance between pulleys. The greater the diameter ratio of the pulleys, the farther apart they need to be in order to get a reasonable wrap angle around the smaller pulley. Unless the pulley wrap angle is reasonable (90 degrees or more), high tension will be required to transfer significant torque. Furthermore, the coefficient of friction also has a great effect, and thus the system must be kept very clean. If excessive belt tension is required, it can lead to shaft overloading. Given these issues, for high torque applications, toothed belts are often used.

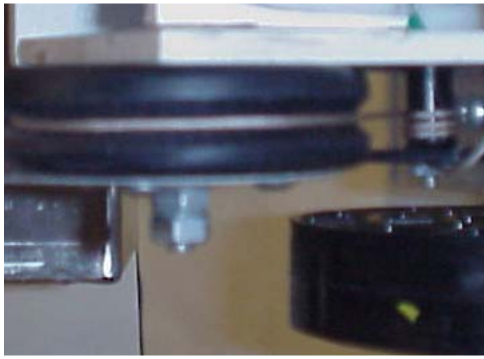
The *accuracy of motion*, or how well the output shaft follows the motion of the input shaft, depends on backlash and belt stiffness. The former almost by definition does not occur in a belt transmission else there would be no torque transmitted. Problems can occur if a toothed belt is used and its tension is too low. When torque is reversed, lost motion (backlash) will occur as the tension in the other side builds up to handle the reversal of load. In addition to the accuracy of motion transferred, the pulleys must be accurately aligned or else the belts will either wander off, or if edge guides are used, excessive edge wear can occur. The primary means of preventing pulley misalignment is good rigid shafts and bearing supports to prevent angular deflection of components subject to belt tension.

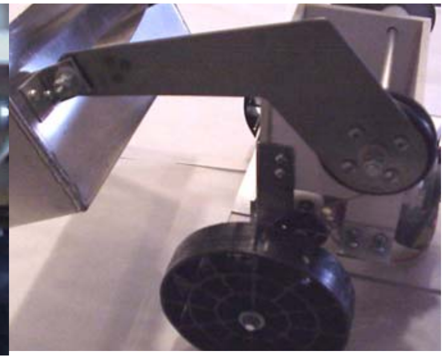
A principle feature of a rotary motion cable driven system is that a cable can bend in three dimensions. Hence the axis of rotation of the motor does not have to be precisely aligned with the axis of rotation of the output shaft. When more torque is needed than a cable wrapped around a pulley (capstan) can supply, a cable with beads molded onto it can be used. A *bead belt*, also enables power transmission to occur in three dimensions. Another example is the use of rubber bands riding in grooved wheels to make a two-wheel drive vehicle into a four-wheel drive vehicle as shown on page 5-14. The rubber bands' stretch provides enough tension to transmit the torque, without requiring the wheels' center distance to be adjustable.

When only partial rotation of a pulley is required to transmit high torque, such as to raise a boom, the ends of the belt (or cable) can simply be anchored on the pulleys. If very limited bidirectional motion is required, the midpoints of the belt can be secured to the pulleys to prevent slip. Look closely at the pictures that show a bucket on the front of a mobile robot. A string is tied to a pulley attached to the bucket, and the other end is tied to a shaft extending from a motor. The string is used as a transmission to increase the motor torque to allow the bucket to be raised. Gravity (it's free!) is used to lower the bucket, and the overall mechanism is greatly simplified.

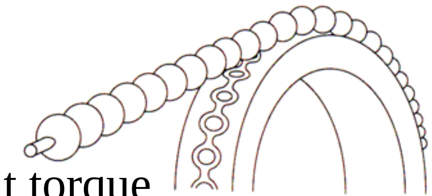
Multiple wraps around the pulleys can enable motion with almost a full revolution and with high torque, accuracy, and stiffness. Commercially, this type of system is available as the *Roto-Lok*™ drive from Sagebrush Technology. This type of cable transmission is used for many types of precision systems where only partial rotary motion is required with high stiffness and no backlash. The key to any cable drive is maintaining cable tension. If the cable is anchored with a clamp, only steel cables will not stretch with time. A spring can be used to maintain constant tension on one end of the cable. The drums often have helical grooves on them in which the cables lay, but for robot contests, smooth surface cylinders typically suffice.

Think of all the rotary motions that occur in your machine (or strategy) and ask yourself how a belt, which would allow a motor to be located away from the actuated component, might make the design more robust or easier to build? Where are you planning on using gears, and how might belts replace them? What would be the physics, cost, risks, and countermeasures? Is the capstan effect (how many wraps do you need?) sufficient for your belts to transfer torque, or do they need to be physically anchored?





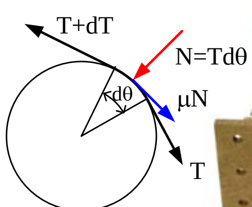
Belts & Cables: *Rotary Motion*

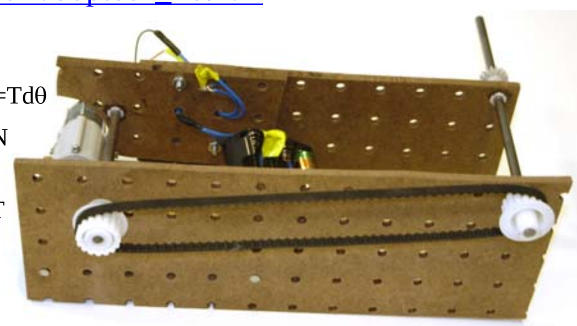


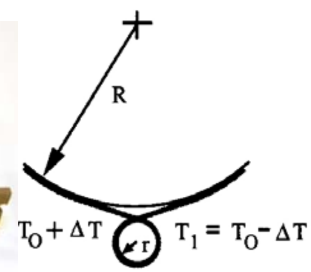
Bead belt drive

- Flat belts and cables (string drives) require higher tension to transmit torque
  - For first order analysis, the belt tension needs equal the desired torque divided by the coefficient of friction and the small pinion radius, BUT CHECK WITH THE CAPSTAN EFFECT EQUATIONS!
- Vee-belts use the the *principle of self help*:
  - Increased tension caused by power being transmitted, wedges the belt in a Vee-shaped pulley groove, so it can transmit more torque…
- *Synchronous Drives (timing, gear, ladder belts)* can transmit torque between shafts and also achieve a transmission ratio
  - They combine the positive timing action of gears with the flexibility, speed and low noise level of belts
    - For an in-depth discussion on synchronous drive design, see Stock Drive Products on-lir http://www.sdp-si.com/Sdptech_lib.htm

Ken Stone, Director of the MIT Hobby Shop, and his string-powered lathe web.mit.edu/hobbyshop







Roto-Lok drive from Sagebrush Technology Inc.

–http://www.sdp-si.com/Sdptech_lib.htm

Belts & Cables: *Manufacturing & Assembly*

There are two primary manufacturing and assembly issues with belts and cables: manufacturing the belt, and assembling the belt and its associated components. Belts are often continuous, although they can be spliced to obtain a desired length. Belts are also often fiber reinforced to give them more strength and to prevent them from stretching with time. The primary function of a belt's rubber or polymer casing is thus to form the gripping surfaces. In robot competitions, however, you may sometimes need to make your own belts from rubber sheet or O-ring material. This can be accomplished by overlapping the material and then cutting through it with a sharp knife. The resulting cuts will match and a suitable adhesive, such as cyanoacrylate (Super Glue™), can be applied. Because these belts will not have a fiber core, they can creep; therefore do not leave tensioned rubber belts on your machine when it is being stored. Nylon string belts can be made by holding both ends to a soldering iron and then forcing the melted ends together.

When assembling a belt system, the pulleys' axes of rotation can never be perfectly parallel, so a flat belt will want to drift off the pulley. This is called *tracking*. to keep a belt from moving off the pulleys, they must be crowned (rounded profile) or one of the axis be made so it can be oriented with respect to the other. The crown forces the belt material on either side to want to climb towards the middle. Neither side can win this tug-of-war, so the system is stable. A flat pulley is at best neutrally stable. On a concave surface, the side with more belt in contact will cause the belt to drift further to that side until it falls off.[1] The crown need not be accurate, and it is easily created on the lathe; however, merely forming a raised central portion on a flat wheel, for example by placing an O-ring on the wheel, does not work. The crown must be formed as a continuous radius.

Cables need a pulley with a groove in its circumference to keep the cable on the pulley. The larger the radius of the groove, the more forgiving of misalignment and the less wear of the pulley caused by the cable as it enters the groove. On the other hand, the closer the radius of the groove is to the cable, the less the cable is compressed under heavy loads. For robot design contests, pulley misalignment is often a dominant manufacturing issue. The figure shows how the Abbe effect dictates the groove radius needed to accommodate pulley misalignment. Conservatively, the groove radius R required to accommodate a cable of diameter d on a pulley of diameter D misaligned by an amount δ is:

$$R = \frac{D\delta}{L} + d/2$$

Tension in the belt or cable can be maintained by stretching the belt by displacing one of the pulleys, using a spring-loaded *idler pulley* (which also should be crowned), or connecting the ends of the belt or cable with a tensioned spring element. Regardless of the method used, being able to consistently set the belt tension to the desired value has a great effect on the repeatability, and hence quality, of a machine. In fact, as shown in the spreadsheet *bandstress.xls* on page 5-6, the tension T in a belt or string can be determined from the measured frequency of lateral vibration ω (Hz), the free length L, the cross sectional area A, and the density ρ:

$$T = (\omega L)^2 A\rho$$

A common example of an idler pulley used to maintain tension is the derailleur on a bike with gears. The tension in the chain from the rider pedaling occurs directly between the sprockets, and one function of the spring-loaded derailleur is to maintain constant tension in the chain even as the ideal length of the chain changes as the rider changes gears.

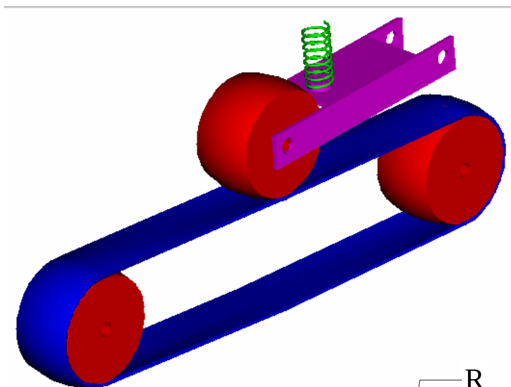
When timing belts are used, they are typically tensioned by moving the bearing assembly that holds one of the sprockets, and then locking it in place. However, the tensile forces from the belt could cause the pulley assembly to become misaligned. In addition, pulling the belt tight with one hand while tightening the holding screws with the other can be difficult. It is better to build the system with a cable tensioning bolt to pull (or push) the pulley assembly so the belt reaches the desired tension.

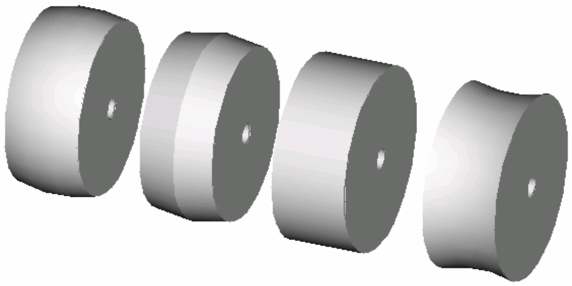
The bane of many a machine with belts is pulley misalignment and improper belt tension. How will your machine establish and maintain belt tension and alignment?

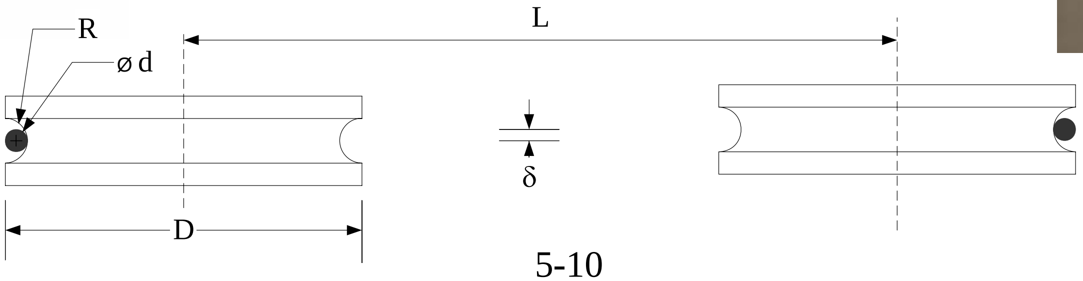
1. Schwamb, P., Merrill, A.L., James, W.H., Elements of Mechanism, Third Edition, John Wiley & Sons, 1921.

Belts & Cables: *Manufacturing & Assembly*

- Misaligned pulleys increase wear and decrease life!
- Two pulleys' axes of rotation can never be perfectly parallel, so a flat belt will want to drift off (tracking)
  - Pulleys must be crowned (round profile) to keep a belt from walking off
    - On a concave surface, the side with more belt in contact will cause the belt to drift further to that side until it falls off
    - A flat pulley is at best neutrally stable
    - Great, OK, bad, & horrid pulleys:
  - Tension must be maintained either by proper pulley center distance and belt elasticity, or a mechanism to tension the pulleys or a pulley that pushes sideways on the belt







Chains

Chains are often used to transmit very large forces and torques relative to their size. There are many different types of chains, and one of the most common types is *roller chain* which uses metal *links* connected together by *pins* and spaced apart by *bushings*. *Sprockets*, toothed wheels which are a special form of gear, positively mechanically engage the chain so there can be no slip. Accordingly, chains are extremely versatile. They can be used to transmit power between two rotating shafts, and they can also be used to convert rotary to linear motion or to enable linear to linear motion. Just take a close look at a forklift, where a hydraulic cylinder provides the force for the first stage to move. How are chains used to move the other stages? Take a careful look at the photos of the FIRST robot whose telescoping stages use chains and idler sprockets to extend them.

As you read the following, try to construct a picture of the design in your mind: *An angle plate attached to two bearing blocks on a first bearing rail is moved vertically by a chain that makes a loop around a drive sprocket on the motor/gearbox and an idler pulley on the top of a first mast. (This first mast is attached to a horizontal motion carriage). A linear bearing rail is bolted to the first mast, linear motion blocks ride on the rail*[1]*, and the angle plate is bolted to the bearing blocks. A chain is attached to the angle plate by bolts that pass through the links. A second mast is also bolted to the angle plate so it can move up and down with the bearing blocks. A second linear bearing rail is attached to the second mast. A linear motion bearing block rides on the second rail and the robot's lifting arm is attached to this bearing block. A chain is attached to the lifting arm and runs over an idler pulley at the top of the second mast and is connected to the top of the first mast.* Now play the movie in your head: what happens as the drive sprocket begins to turn?

Another versatile aspect of chains is that they can be laid onto a surface, and anchored at the ends to act as a gear rack (page 6-23); in addition, a chain can be held to a curved surface to act as a gear. As shown in the photo, a chain was used as a rack for a FIRST robot. Note how the sprocket was welded to a clamp-type collar and then mounted on a motor/gearbox shaft so the force generated at the sprocket teeth by the motor/gearbox torque creates a reaction force on the shaft that passes directly through the shaft-support bearing and thus will not harm the motor/gearbox: Motor gearbox shafts are typically only meant to transmit torque and have very limited moment capacity. Hence this design eliminates the need for a separate support shaft, bearings, and coupling.

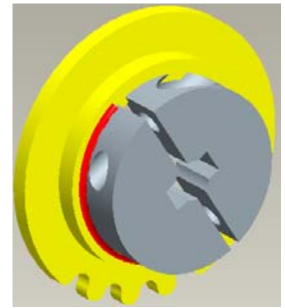
Sprocket center distance and tension in the chain can be determined using the same calculations as used for belts. With regard to acceptable loads in chains, manufacturers provide recommendations for different sizes and numbers of strands. Like belts and cables, tension must be set or maintained; however, because the teeth on the sprockets are relatively large and protrude nearly all the way through the chain, chain drives rarely skip and fail to transmit the desired torque. For bi-directional motion, lack of proper chain tension will result in backlash (lost motion or hesitation) when the motion is reversed. Chain tension can be set using an idler pulley on a spring loaded arm, as is done for belts, or by making one sprocket movable so the tension can be set and then the sprocket locked into place. Idler pulleys are often mounted at one point and are used to direct the motion of the chain, such as for extending linear axes. Idler pulleys can be purchased that have integral ball bearings with their hubs, and then a bolt, e.g., a shoulder bolt, can be used as a non-rotating axle to support them.

Sprockets are readily available in many different sizes. A chain breaker tool is used to press out pins from bushings so the chain can be made the proper length and then rejoined using master links. Various special types of links can also be obtained so other elements can be more readily attached to the chain. Cutting teeth can also be attached as in a chain saw. All are readily available from catalogs.

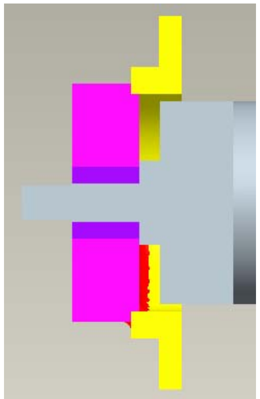
Look at the chain, sprockets, and shifting system on a bicycle. How does the system accommodate the different length of chain required to go around different sets of sprockets? Look at a forklift, how are chains used? What other devices use chains? Can a chain, driven by one sprocket, be used to drive multiple sprockets? Could a chain be used as a crawler track on a robot?

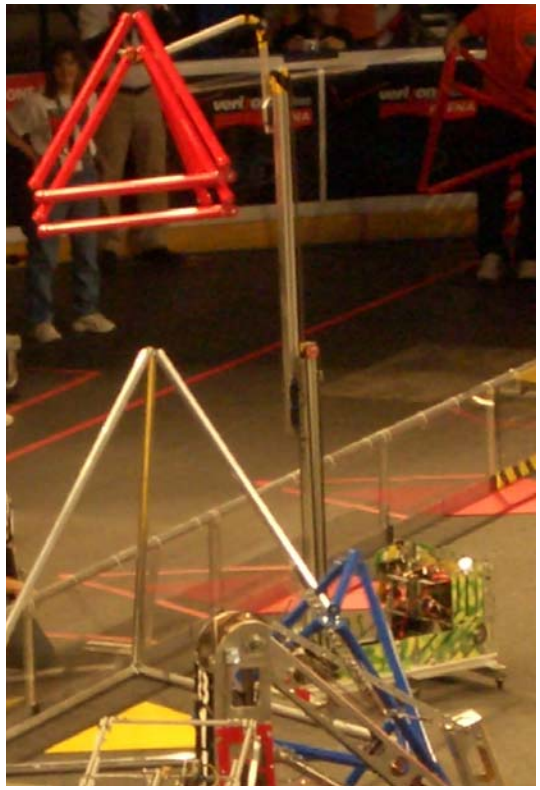
1. See page 10-12

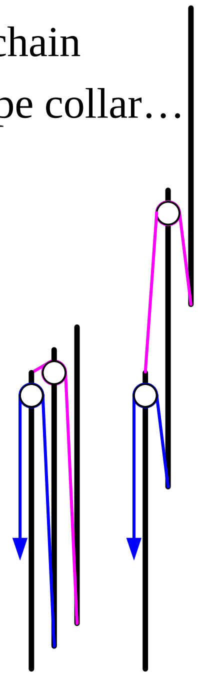
Chains

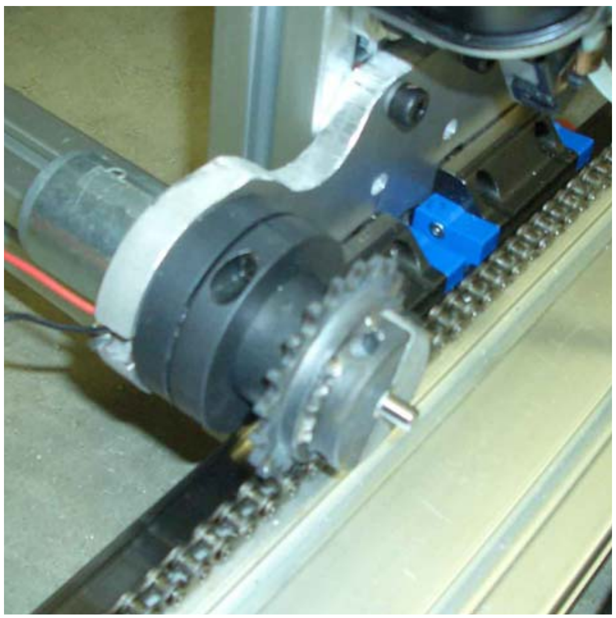


- Chains are a very robust means to transmit very large forces and torques
  – Rotary-to-rotary motion is very common (look at your bicycle!)
  – Rotary-to-linear motion can be obtained by laying a chain on a surface, and anchoring each end with bolts through the links
    - A sprocket mounted to the motor "crawls" along the chain
    - The sprocket can be welded or bolted to a squeeze-type collar…
  – Linear-to-linear motion for extending axes
    - Idler pulleys are invaluable

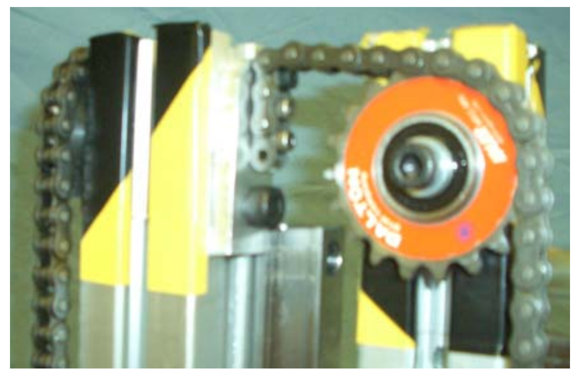








Bishop Brady High School's rookie FIRST robotics team used chain drives to realize a very simple and effective robot that earned the *Rookie All Star* Award and *Top Rookie Seed* awards at the 2005 Manchester regional competition and a trip to the finals in Atlanta.



Chains: *Engineering*

It is a simple task to design a system that uses roller chains to ensure that they can carry intended loads for the life of the chain. Roller chains' bearing contact interfaces between the rollers and pins act as mini shock absorbers, but their load capacity needs to be decreased by a $k_{service\ factor}$ that corresponds to the type of system and load they experience[1]:

$k_{service\ factor}$	Input power		
Loading	Hydraulic Drive	Electric Motor	I.C. Engine & mechanical drive
Smooth	1.0	1.0	1.2
Moderate shock	1.2	1.3	1.4
Heavy shock	1.4	1.5	1.7

Single row chain is very robust to misalignment of sprockets, because the sprocket teeth protrude into the chain it cannot wander off the sprocket the way a belt can wander off a misaligned pulley. Multiple strand (row) chain is used to transmit very large forces with a low profile, or to provide redundancy should one link fail. With each added row, misalignment capability decreases. Merely adding a second row to a chain does not double its load capacity:

Number of Strands	Multiple Strand Factor	$K_{strand\ factor}$
1	1	1.00
2	1.7	1.18
3	2.5	1.20
4	3.3	1.21

Given an applied chain tension *T*, be it from a linear or rotary motion application, an equivalent tension has to be determined to reflect the type of load and number of strands of chain required. An equivalent tension must be below the acceptable limits for the particular chain used, as specified by the manufacturer in order to obtain the desired life:

$$T_{\text{equivalent}} = T_{\text{applied}} \times K_{\text{service factor}} \times K_{\text{strand factor}}$$

1. ASME Standard B29.1M-1993

The size of the roller chain, in the USA, is referred to as the *Chain Number*[2]. The *pitch*, distance between the rollers' centers, increases with chain number, as do other characteristic dimensions and load capacity:

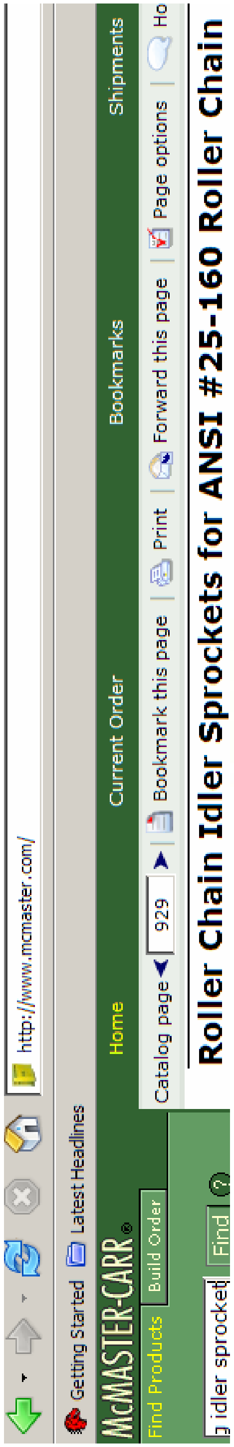
ANSI Steel Roller Chain (units of inches and pounds)							
Chain #	Pitch	Roller		Pin Diam.	Link plate thickness	Breaking Load	Working Load
		Diam.	Width				
25	1/4	0.130	0.125	0.091	0.030	1050	140
35	3/8	0.200	0.188	0.141	0.050	2400	480
41	1/2	0.306	0.250	0.141	0.050	2600	500
40	1/2	0.313	0.313	0.156	0.060	4300	810
50	5/8	0.400	0.375	0.200	0.080	7200	1400
60	3/4	0.469	0.500	0.234	0.094	10000	1950
80	1	0.625	0.625	0.312	0.125	17700	3300

Some practical use guidelines include:

- The total length of chain loop should not exceed 100 pitches
- Number of sprocket teeth: <17 low speed, 17-21 moderate speed, >21 high speed.
- Due to the manner in which the rollers engage the sprocket teeth, assuming the driver sprocket is rotating at a constant speed, the driven sprocket speed will vary by $\cos(2\pi/N_{\text{teeth on the driven sprocket}})$
- Number teeth large sprocket/Number teeth small sprocket < 7
- Noise increases with increasing Load/Maximum Load
- At least 1/3rd of the teeth on a sprocket should engage the chain
- Idler sprockets can be used to maintain tension
- Lubricated chain and sprocket life can be long because the teeth nominally have rolling contact with the bushings, although there is enough slip to require lubrication.
- Use protective covers to keep stuff out of the chain drive!

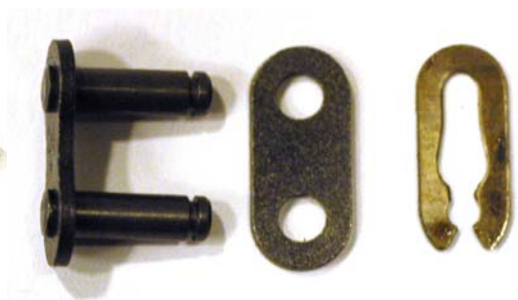
Note the factor of 7 difference between the working load and the breaking load for a chain. Determine the size of the chain on your bicycle and based on the crank length, determine how hard you could push on a pedal before you snapped the chain on your bike. How strong are you?

2. Size 25 and 35 chains' rollers do not roll on the bushings.

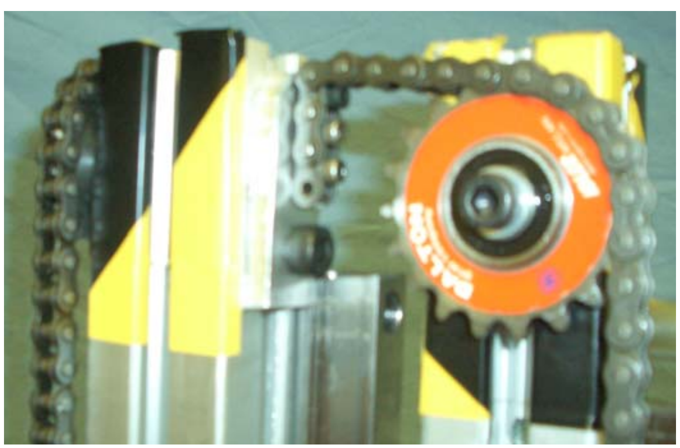


Chains: *Engineering*

- There are many different components that designers can use to create systems
- A *chain breaker* (a press) pushes out the roller chain's pin
  - Master links are used to splice chain ends together to form desired length loops
    - Master links are available with fittings to allow things to be attached to the chain


- Idler pulleys allow for routing of the chain
  - Idler pulleys can have integral ball bearings
  - A bolt through the center supports and anchors the idler pulley



Wheels

We can only guess the first ancient human thought who perhaps saw a rock rolling down a hill. Did they think that since a heavy rock could roll, maybe other heavy things they were carrying could rest on top of the rock? Or maybe Og stepped on a log which rolled out from under him? Or maybe Ogette was stacking firewood and the stack became too high and started to roll apart? Maybe she observed a tree falling on top of two other tress and then the assembly rolled away? Whatever the origins, there are several important lessons: We are a race of builders who takes advantage of what others have done; and we are a race of observers who learns by observation and experimentation. Consider some of the typical functional requirements for a wheel, and think about the design parameters and physics that come to mind.

Table 3:

Functional Requirements	Design Parameters	Analysis
High speed	D diameter (large)	$V_{vehicle} = 2\omega_{motor}/D_{wheel}$
Climb over obstacles	D diameter (large)	$F \geq F_M \dfrac{\sqrt{Dh - h^2}}{D/2 - h}$
Controllability	Wheel size, steering linkage	Larger wheels go faster, may be more difficult to control
Traction	Materials in contact, vehicle weight distribution	$F_{traction} = \mu F_{normal}$ $F_{traction} = 2\Gamma_{torque}/D_{wheel}$
Easy attachment	Flange, spline	Torques and moments

What other functions do wheels have? Why are bike tires not hard rubber so you would not have to worry about them going flat? Why are they not filled with foam rubber? Why is it easier to ride a bike with big wheels than with little wheels? And why are little kids bikes not designed with big wheels? In addition to bicycle wheels, what are the kinematics, dynamics, and economics of wheel-based vehicles? What are the risks and countermeasures associated with each design choice?

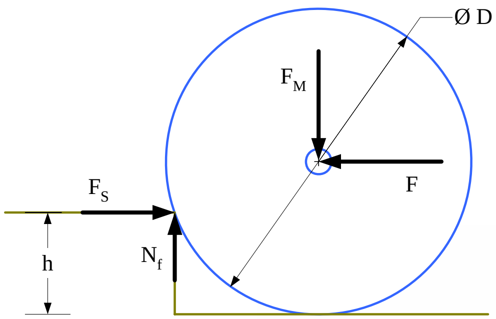
The *kinematics* of a wheel can be determined by realizing that the instant center is the contact point between the wheel and the ground. In the case of a wheel encountering an object, the instant center is at the contact point with the object (e.g., a curb). When a vehicle is supported by many wheels, as is typically the case, the primary issue is the alignment, or in some cases, purposeful slight misalignment so as to help bias a high speed vehicle against spin outs. When a car turns, the linkage must adjust the angles between the wheels and the car body, but also the relative angles between the wheels may be adjusted because each wheel takes a separate path through the turn. When the wheels on your car are misaligned, the result is typically uneven wear and higher fuel consumption as well as the potential for poorer handling. In a robot contest, misalignment can result in lost tractive capability and poor controllability.

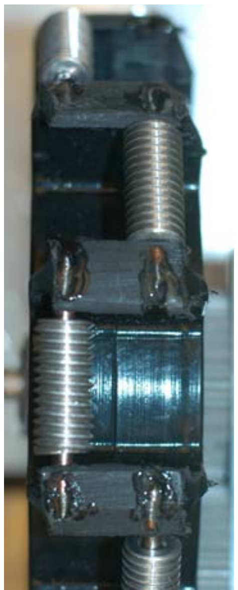
The *dynamics* of wheels are principally a result of their angular momentum. Large wheels' moments of inertia increase with the 4th power of their diameter (3rd power for a thin rim spoked wheel), and this directly affects the torque required to get a wheel up to speed. Often the reflected inertia of a large wheel (see page 7-4) can be greater than the inertia of a vehicle itself! What about angular momentum? Can it help or hurt the performance of a vehicle? What about the speed of a wheel on a surface and the potential for entrapment of air or liquid (hydroplaning)? How can it be prevented?

The *economics* of wheels are terrific in that they are simple to design, manufacture, and implement because they have so few parts compared to, for example, a track system. Their simplicity is perhaps how/why they were discovered ages ago and have been in use ever since. However, for specialized applications, even though they may be fundamentally simple, the details are what matters. For example, adhering a layer of sandpaper or Velcro to a plastic wheel to greatly increase its tractive effort on a soft surface such as carpet.

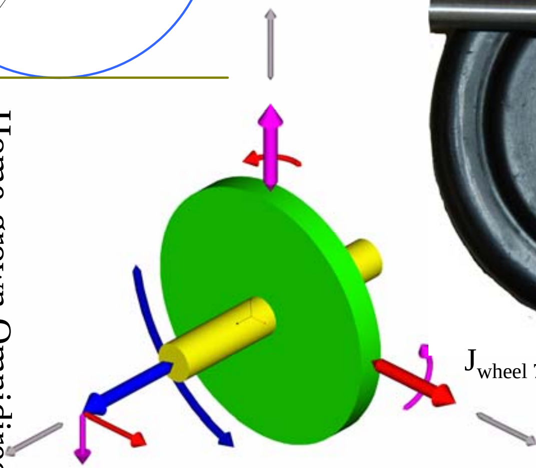
Wheeled vehicles are very simple to design and build and they work well on even surfaces, so make a simple vehicle and test it on the contest table as soon as possible. Would you be better off with a tracked vehicle? Can a wheeled vehicle be made to skid-steer so it can turn in place? What makes street sweepers so agile? What benefits can your robot obtain if it is highly agile? What are the trade-offs to using one motor and control channel to control the steering system?

Wheels

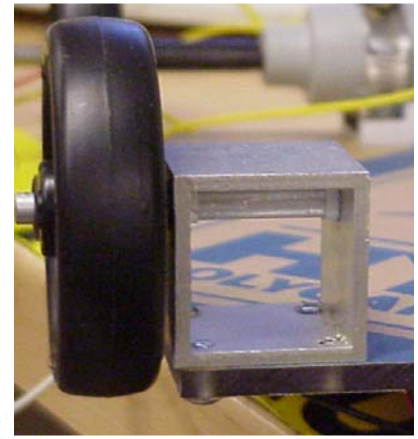




Home-grown Omnidirectional wheel



$J_{\text{wheel 75 mm diameter}}$ = 7.1e-5 kg*m^2



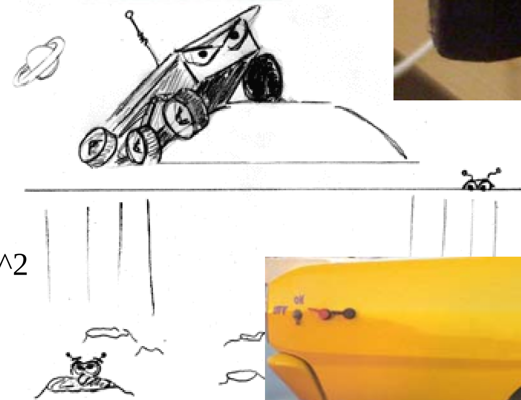
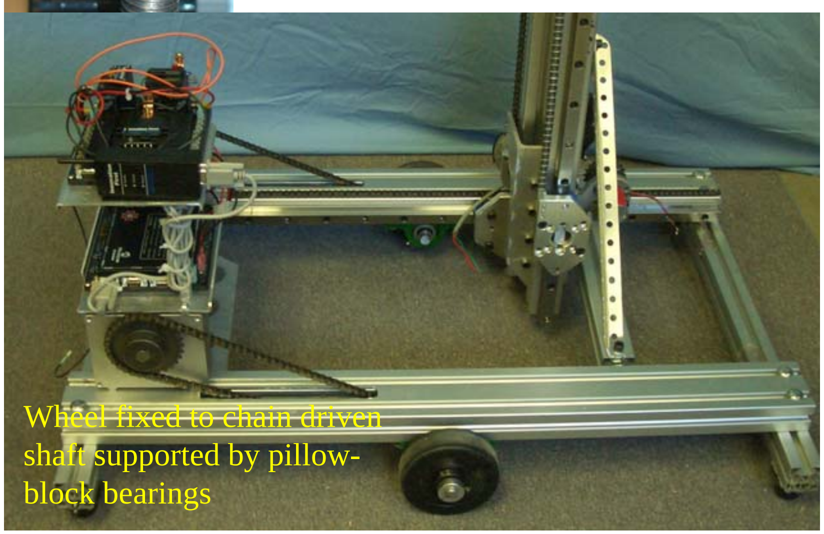
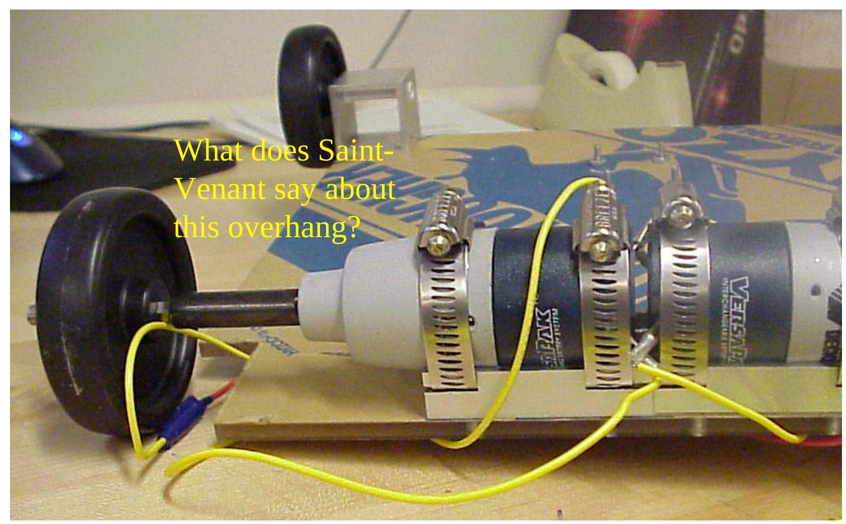


Photo by Rick Slocum www.100jpegs.com

- Traction & Controllability
- Size, Torque & Contact Pressure
- Manufacturing & Mounting





Wheels: *Traction & Controllability*

Traction is the ability of a wheel to turn without slipping and this directly affects the force that the vehicle can exert on its own mass and on other objects. Controllability is the ability of the wheel to steer the vehicle. Steering can be done using a linkage and is affected by how the wheel makes contact with the road as it is turned. Steering can also be in the form of pure traction if skid steering (differential speed between the wheels) is used. *Without good traction a vehicle will have poor controllability*[1]. Consider front wheel drive cars, which have better traction than rear wheel drive because more of the car's weight is over the front wheels (unless the trunk is filled with Scuba gear!) If tractive forces are being applied that are nearly all of the total tractive effort, will there be enough tractive effort left for the steering effort?

Steering a fast moving vehicle, such as a bicycle going down a steep hill, can be easier than steering a slow moving vehicle. The angular momentum principle requires that the torque vector in a system be perpendicular to the time rate of change of angular momentum. If a wheel is spinning about an axle at a high rate of speed, and you attempt to rotate the wheel about another axis, a torque on the system will be produced about the third perpendicular axis. If you were to sit in office chair that can swivel while holding horizontal a spinning disk, when you tilt the spinning disk, your chair will start to swivel. When the spinning disk is supported in a *gimble*, a frame that allows the disk to pivot about two axis orthogonal to the spin axis, it forms a *gyroscope*, or *gyro*. From this basic principle, *rate gyros* and *control gyros* evolved: A rate gyro's support axes are free to spin, and sensor's measure their orientation. By observing the orientation of the support axes, one can deduce the rotation rate (angular velocity) of the system that holds the gyro. Conversely, a control gyro uses torque motors to apply torques to the gyro's support axes, and the resulting reaction torques on the structure cause it to move with respect to the spinning gyro. How might robots use these principles?

Traction is important for steering and for moving forward, and a primary design parameter for enhancing traction is the normal force between the wheel and the surface. How does driving up an incline[2] affect the traction of a two-wheel-drive vehicle verses a four-wheel-drive vehicle? Does front-wheel-drive hurt or help? How else can the normal force be increased? Another primary design parameter for enhancing traction is the effective coefficient of friction between the wheel and the surface. In many engineering applications, surface finish does not affect friction, but in the case of a wheel running on a rough surface in the presence of dirt, treads can help to push aside debris and help prevent hydroplaning. In the case of a soft surface, treads or other features can actually engage features on the surface to create an effective coefficient of friction greater than 1.

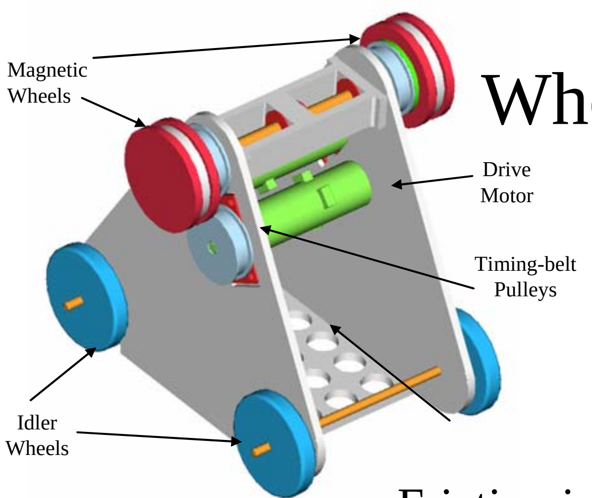
Wheels can also be used to drive components, and in this application, they are commonly known as friction drives. Perhaps the most common friction drive is the use of wheels to control the motion of paper in ink jet printers. Are the wheels smooth rubber, or do they have tiny surface features that engage the micro rough surface of the paper, thus driving it like a gear rack? Coordinate Measuring Machines (CMMs account for 10% of all machine tool sales) often use friction drives to provide an extra margin of safety. If a person gets in the way, the friction drive wheels will slip before can hurt the user.

The primary design challenge with friction drives is that the preload force must be greater than the desired drive force/coefficient of friction! In addition to preloading one roller against another, magnets can be used to preload the wheels against the ferritic steel surfaces. This idea has been invented many times by designers who have created inspection robots to drive along metal pipes. If magnets can preload the wheels to the surface, then the same magnets can preload a small wheel attached to a motor to the magnet wheel, thus creating a friction drive transmission between the motor output and the wheel. This is the concept for the *Magnabot* robots for hospital automation designed at MIT[3]. The goal in this case was to create robots that could drive on ceilings in halls and stairwells to automate delivery functions in hospitals.

Plan ahead for steering and traction. When you get your kit of parts, play with the kit materials to evaluate their relative friction coefficients, and their friction coefficients on the contest table surface! What wheels are in the kit, and what other materials do you have that might enable you to make even better wheels?

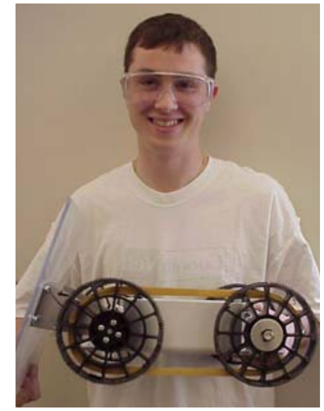
1. In addition, in robot contests in particular where humans often control the robots, if the wheels are too big, then a small signal from the controller will send the robot zooming off into a barrier.
2. Look ahead to Chapter 8 for a detailed discussion of system stability

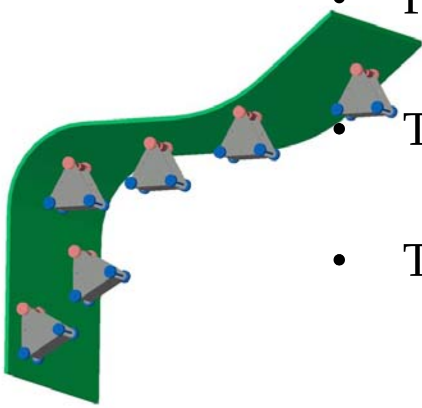
3. http://pergatory.mit.edu/magnabots



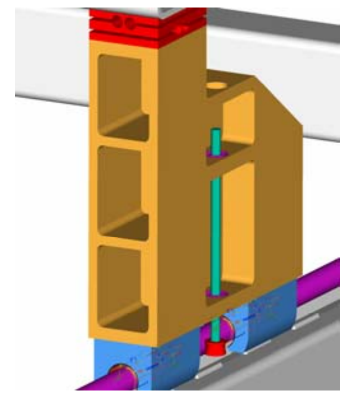
Wheels: *Traction & Controllability*

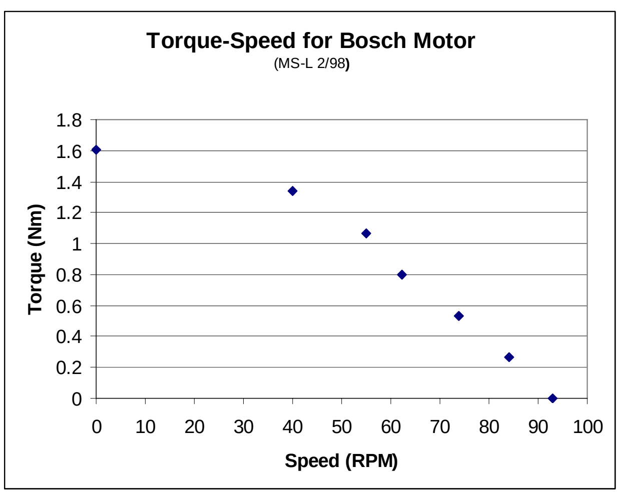
Nick Powley obtained 4WD by cutting grooves in his wheels and using rubber bands as drive belts…





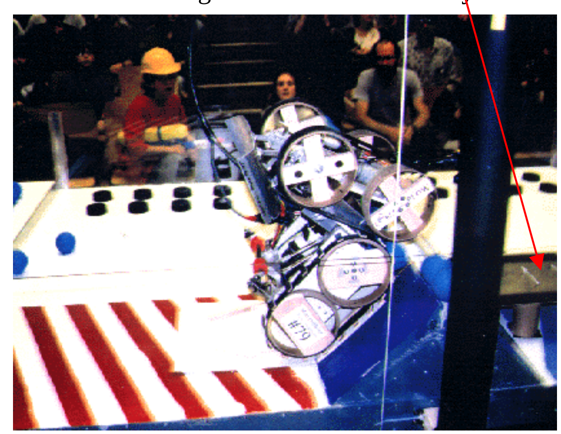
- Friction is independent of surface finish, unless there is physical engagement
  - Like gear teeth on a rack, sandpaper grips carpet, plastic…
- Two-Wheel Drive vehicles are simple to build
  - Against a 4-Wheel Drive or a Tracked vehicle, they lose
- The smaller the wheel:
  - The greater the traction force the motors can cause
  - The slower the vehicle per unit input to the controller!





Tom Slowe made a very successful 2WD vehicle with sandpaper covered rear driven wheels

Tim Zue's sandpaper covered platform enabled him to get to the other side every time!



Wheels: *Coefficient of Friction*

A very common question faced by design students building contest machines is how to obtain a high coefficient of friction between the wheels and the surface? Various methods tried over the years including gluing rubber bands to plastic wheel surfaces, and screwing sharp objects into the wheels to create spiked wheels. The former is not too effective, and the latter is extraordinarily dangerous. In fact, sharp-spiked wheels are like spinning saw blades and can shred skin. Never make sharp-spiked wheels!

What about treads and how to make them on the surface of plastic wheels? Treads are generally useful on soft ground where they can dig in and form interlocking features which then shear before slipping. Plastic wheels interestingly enough can have treads put on them by mounting them in a lathe and using a knurling tool to form the tread pattern on the surface; although a knurling tool is usually used with metal objects, it also works on plastic wheels!

A compromise on the spiked surface is to glue coarse sandpaper onto the surface. The prime challenge then is what adhesive to use, and what type of joint should be used between the ends of the sandpaper. The wheel surface should be scuffed up with sandpaper, and cleaned with alcohol before the epoxy is applied. The sandpaper can be cut to “exact” length to form a butt-joint, and the joint filled with epoxy. Rubber bands or tape can be used to keep radial pressure applied until the epoxy sets. As long as the sandpaper does not overhang the edges of the wheels, there should not be free edges to catch and peel.

So which type of tractive surface should be used? What are the risks and countermeasures associated with each? The simplest solution for the problem at hand is the best, including predicting what happens when you get into a pushing contest with the other contestant. Do you have enough torque and friction to climb over obstacles, as discussed on the next page?

The pictures show different wheel surfaces, and a picture is also shown of a simple bench level experiment to determine the coefficient of friction for each surface on different surfaces. Each of the tests was performed 11 times and the high and low values were discarded. The average of the 9 readings then meant that random errors would be reduced by a factor of 3. The cart was placed on different positions on the test surfaces, and the variance was less than 10%. An interesting observation is that the 80 grit sandpaper grits seemed to interlock with the carpet fibers, so the test cart never slid, but merely tipped over. hence the value for the coefficient of friction of sandpaper on carpet is not accurate.

Designers need to be aware that if they are creating a four-wheel-drive machine that steers by driving wheels on one side of the machine forward and wheels on the other side backwards, a *skid-steered machine*, that the wheels have to slide sideways. Hence the coefficient of friction which helps when driving forward hurts when trying to turn. If the machine has enough torque, it can overcome the resistance, but control can be difficult. A bench level experiment is warranted early on!

If designers are creating a two-wheel-drive skid-steered machine, then the only detail to remember is that the non-driving wheels should have a very low coefficient of friction between them and the surface. This enables them to slide sideways easily when the machine is steering, and it has no effect on the tractive effort of the machine.

How would you obtain an “accurate” coefficient of friction” test for the sandpaper covered wheels on a carpeted surface? What is the coefficient of friction of a rubber band covered wheel on a plywood surface? On a carpeted surface? Would making many small radial slits on the wheels be better than the knurled surface? To minimize cost (time) and risk, should you just go ahead and cover your motor-driven wheels with sandpaper? If you have a four-wheel-drive car, make sure to do a quick bench level experiment to make sure your motors have enough torque to actually skid steer the car. You may have too much friction to steer!

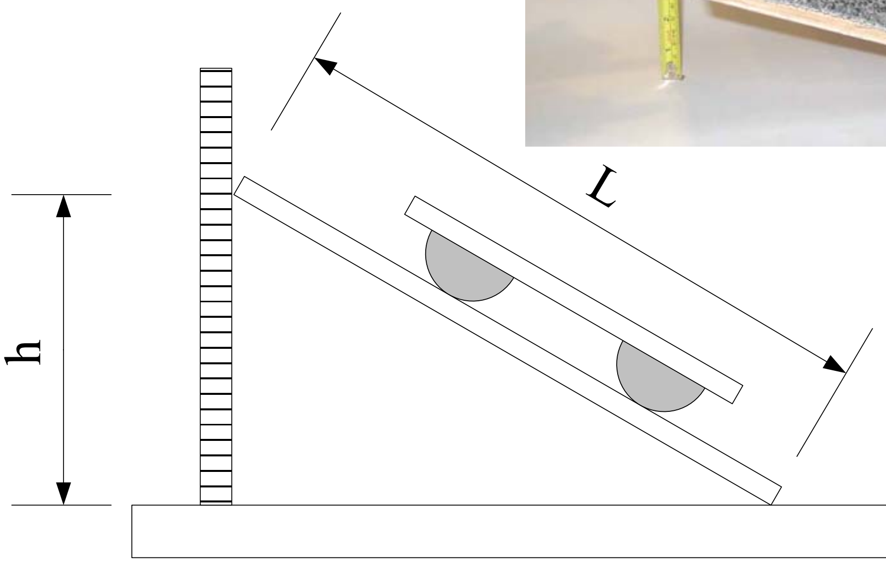
Wheels: *Coefficient of Friction*

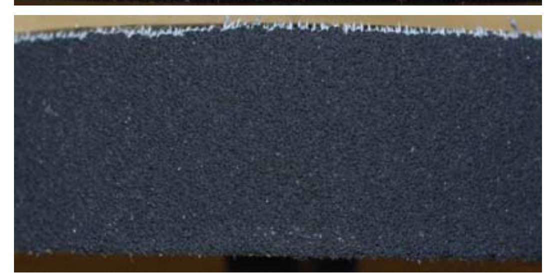
- Simple inclined plane tests were done
- Hypotenuse length = 400 mm
- Raise end of ramp till test car slips:

$$\mu = \tan\theta = \frac{h}{\sqrt{L^2 - h^2}}$$

	Surface		
Coefficient of friction	Plywood		Carpet
Wheel surface	along grain	across grain	
Plain	**0.45**	**0.47**	**0.31**
Knurled	**0.47**	**0.48**	**0.64**
Sandpaper covered	**0.91**	**0.92**	**1.81**

  – The sandpaper covered wheel tester on carpet did not actually slip, it tipped!





Wheels: *Size, Torque, and Contact Pressure*

The primary design parameters for a wheel are its size and the tread material. The former affects the overall transmission ratio of the machine and the size of the object that the wheel can climb over, and the latter affects the coefficient of friction and hence the achievable traction force that can be obtained. The width and the shape of the contact surface, may also be of importance depending on the application. For example, when driving on soft surfaces, a wide compliant wheel may be desirable to minimize sinking into the surface. When driving in a hard surface, a narrower wheel has lower frictional losses caused by the fact that the contact area effectively means that the wheel is rolling simultaneously with two different radii in contact with the ground.

In order to determine how big a wheel must be in order to overcome an obstacle, the sum of the moments about the contact point must equal zero: the tractive effort F moving the vehicle forward must create a moment to overcome the force F_M acting downward through the center of the wheel:

$$\sum M = 0 = F\left(\frac{D}{2} - h\right) - F_M\sqrt{Dh - h^2}$$

$$F \geq F_M \frac{\sqrt{Dh - h^2}}{\frac{D}{2} - h}$$

The tractive force F that is applied depends on whether the vehicle is a two or four wheel drive vehicle. In the former case, the back wheels are pushing the vehicle forward, and the maximum tractive force that can be obtained is a function of the motor torque Γ, wheel diameter D, normal force F_N, and coefficient of friction μ:

$$F = \frac{2\mu F_N \Gamma}{D}$$

What happens when a four wheel drive vehicle is used? The front wheels engage the obstacle and quickly lose contact with the ground. Their tractive effort is dependent on the motor torque, wheel diameter, coefficient of friction and the tractive force from the rear wheels pushing them against the obstacle. Unless the coefficient of friction is near 1, which is rare, having four wheel drive does not necessarily enable the vehicle to twice as easily climb over obstacles. However, once the front wheels make it over the object, what happens to the rear wheels when they engage the object? In general, the efficiency of a four wheel drive vehicle in climbing over obstacles is equal to the coefficient of friction between the wheels and the road, such that the effective obstacle climbing tractive force a 4WD vehicle with equal weight distribution on its wheels can achieve is typically $(1 + \mu)$ times the tractive effort of a 2WD vehicle.

The analysis shows that typically the force required to climb over an obstacle drops in proportion to the ratio of the height of the obstacle to the wheel diameter. Thus it is no big surprise that bigger wheels can more easily climb over objects. But how does the use of a bigger wheel affect the required motor torque?

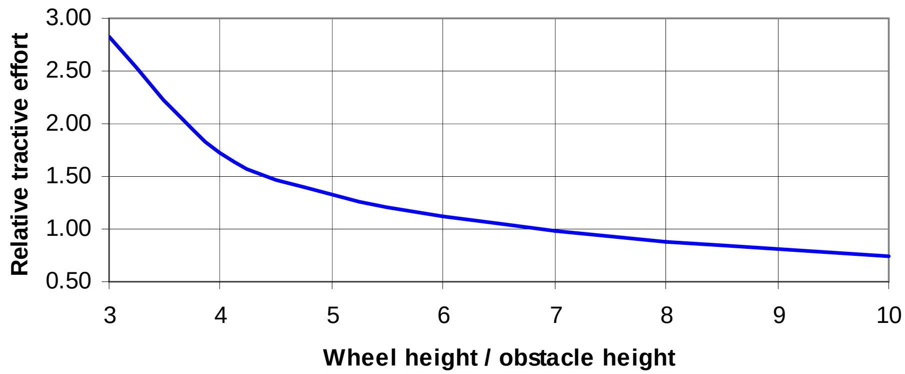
What about contact pressure and the coefficient of friction between the wheel and the ground? If the contact pressure is too high, the wheel or the ground may permanently deform. This is very unlikely in any robot contest unless the table purposefully has a very soft surface. But does it help the tractive effort to have a high contact pressure? In general, unless there are tread features that physical deform and engage the surface, the effective coefficient of friction will not be affected by the surface contact pressure.

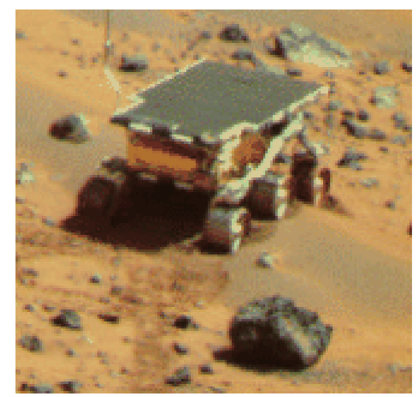
For hard wheels on a hard surface, such as nylon wheels running on metal that act as a linear motion bearing, it is necessary to determine the contact stress. This can be done using Hertz Contact Stress theory as described in detail in Topic 9. Unless a very compliant wheel is used, it is very difficult to obtain contact across all the widths of rectangular profile wheels that support a vehicle. To avoid loading and wearing the edges of the wheels, a slight crown is typically used.

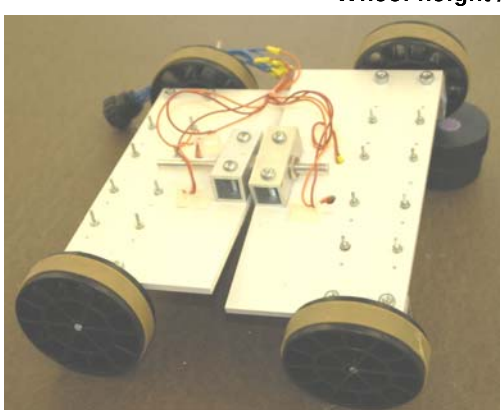
Experiment with *WheelClimb.xls* and you might be surprised at how low an object a 2WD vehicle with modest coefficient of friction can climb! Nylon wheels on plywood may only have a coefficient of friction of 0.1. No wonder many a student who did not bother to do the calculations was stunned to see their great machine spin its wheels when faced with low barriers! What can you do to increase the coefficient of friction between the wheel and the table? How can you easily achieve 4WD or increase μ so 2WD is sufficient?

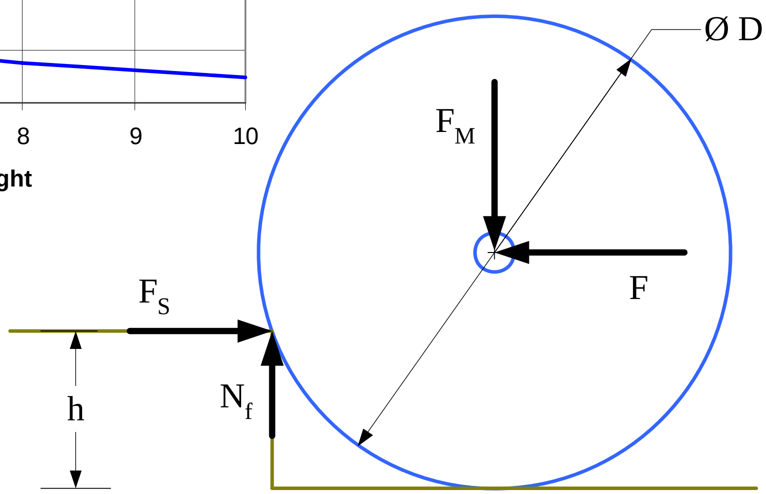
Wheels: *Size, Torque and Contact Pressure*

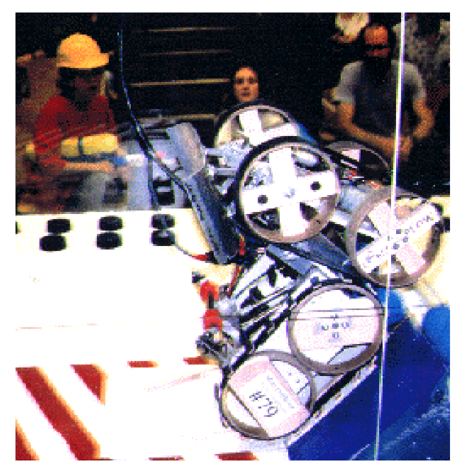
- How big must a wheel be to make sure it can overcome obstacles?
  - The smaller the wheel diameter, the lower the torque to turn it
    - Ideally, the wheel just slips when maximum torque is applied
      - This keeps you from stalling the motor and potentially burning it out
  - The smaller the diameter, the higher the contact pressure, and the greater the wear
    - Hertz Contact stress theory can determine if wheels are too heavily loaded!
    - The bigger the wheel, the easier it is to drive over an obstacle
    - See *Wheelclimb.xls*











Wheels: *Manufacturing & Mounting*

There are four basic steps in making and mounting wheels to a robot vehicle for a design contest: 1) selecting the material of the wheel for use with the contact surface, 2) making (or obtaining) the wheel, 3) mounting the wheel to a shaft such that it either spins freely on the shaft, or torque is transmitted without slip between the shaft and the wheel, and 4) making sure that all the wheels are properly aligned so that the vehicle efficiency is maximized.

Selecting the proper wheel material is very important. Some wheels are made of plastic because they are designed for use on carts that are pushed, so their coefficient of friction with the floor material is not important. However, for a robot contest, a low coefficient of friction is bad; thus if you are given plastic wheels, or plan on making a plastic wheel, you may want to adhere a layer of rubber or sandpaper (if allowed) to the outside rim of the wheel to increase its coefficient of friction with the ground. If the ground is soft, you may even want to consider making the wheel have teeth, much like that of a fine pitch gear, so as to help it obtain better traction.

Assume the wheel diameter has been chosen, for example as discussed to ensure the vehicle can overcome an obstacle. If a suitable wheel is available as an off-the-shelf component, use it! Otherwise you can easily make a wheel by cutting material out of a sheet of flat stock. Remember, in most circumstances for robot design contests, the width of the wheel does not matter when driving on a hard or a carpeted table. Beware, that if you are skid steering your vehicle on a carpeted surface and the wheel is too thin, the wheel can get stuck in its own groove and the vehicle will not be able to turn.

The width of a wheel must also be sufficient to prevent it from folding in half. In the limit, a piece of paper will fold over sideways if you try to use it as a wheel. Simple beam bending calculations can be used to estimate the axial strength of a wheel, or you could do an FEA, or a simple test. What is more likely to be an issue is the effect of wheel width on its connection to a shaft.

If the wheel is to run true and not wobble as it rolls, then the required width of the mounting region can be calculated. Assuming the mounting tolerances and the allowable axial wobble at the wheel rim, the wobble is simply a sine error. For a high speed system, an engineer would create a spreadsheet or program to calculate the wobble. They would then determine the moment put on the shaft that would be caused by the changing angular momentum vector of the wheel. In this manner, the imbalance forces, and hence the vibration in the system, can be determined. Given an allowable level of vibration, the engineer can determine how accurately or over what size region the wheel needs to be mounted.

For a typical robot design contest, where dynamic balance of the wheel is probably not an issue, Saint Venant's principle can be used as a design guideline. Assume that if the wheel is of radius R, then if the length of engagement on the shaft is less than R/5, the wheel is likely to dominate the shaft and we are likely to get excessive wobble, even for a robot contest. On the other hand, an engagement ratio of 1:1 is probably unrealistic. If we try to engage the wheel on the shaft with a characteristic dimension of about R/3 or less, acceptable performance will be obtained.

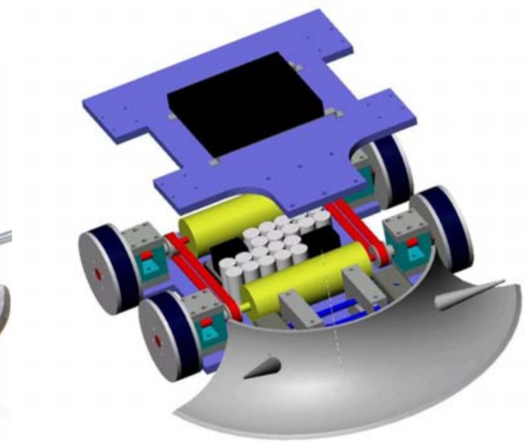
If a wheel is to be driven by a motor, it must be securely attached to the shaft. The best way for this to be done is by using a shaft with an integral hub onto which the wheel is bolted, just like on a car. The next best thing is a spline connection that can be nearly achieved with a hexagon shaft and a wheel that has been broached to have a hexagon hole in its center. If this is not possible, then the wheel should be keyed to the shaft. If this is not possible, then the wheel can be pinned to the shaft; however, take care that the pin and hole are located on the outside of the wheel such that wheel loads that cause bending of the shaft do not place the region of the shaft with the hole in bending. As a last resort, press fit the wheel onto the shaft. These same mounting ideas apply to gears or pulleys on shafts.

The last item is alignment of the wheels, and this typically involves making sure the axis of rotation of the front and rear wheels respectively are parallel. This is easily accomplished by means of a fixture, which can be a straightedge that is placed along the wheels' faces. However, this does require that the bearings that support the axles to be mounted in an angularly adjustable fashion. Many students are indeed surprised when they put together their robots and they have trouble controlling them because the wheels are so poorly aligned. In the case of tracked vehicles, poor misalignment can cause belts to come off even highly crowned pulley wheels.

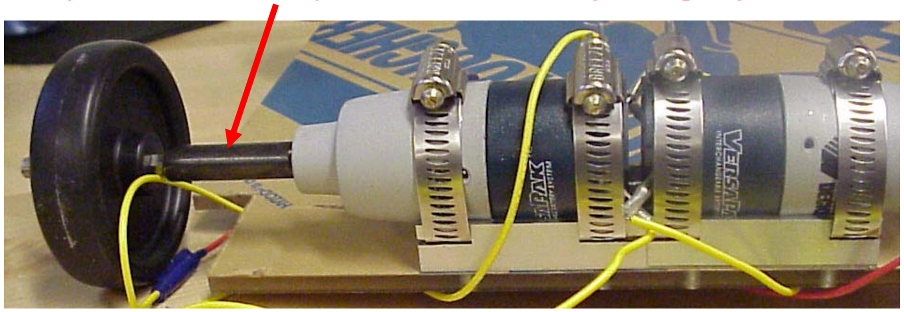
Have you carefully designed your wheels, connections and alignment capabilities? Now would be a good time to check!

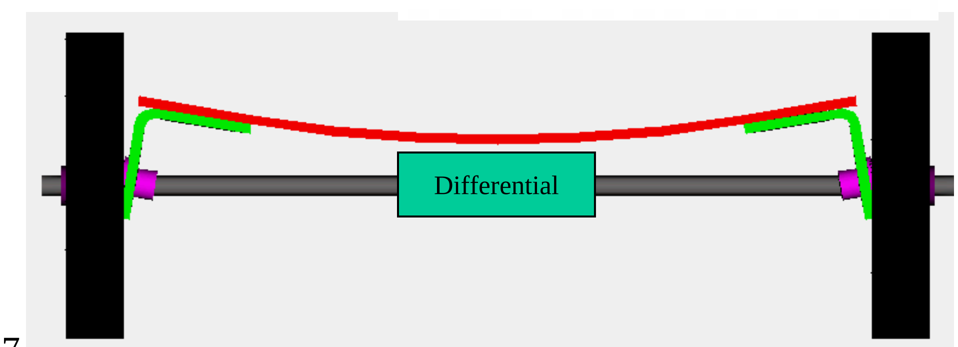
Wheels: *Manufacturing & Mounting*

- A positive engagement is required to effectively transmit torque from a shaft to a wheel
  - A hexagon hole can be broached into pulleys or wheels and square or hexagon shafts can be threaded!
  - Press-fits, keys, or pins also transfer torque, and E-clips are a great way to retain thrust washers
  - In production, a three lobe polygon is the best interface (see DIN standards)
- A wheel-on-shaft subassembly should overhang the gearbox as little as possible!
  - If the overhung shaft has a lot of compliance, an "outrigger" bearing can be added without over constraining the shaft, if it confines the shaft to its nominal central position
- Beware of system deformations and the fact that they can take up all the clearance in a system and cause it to bind



Too much overhang (remember Saint-Venant!). Consider the use of an outrigger bearing, or mount the wheel on a properly supported shaft and drive the shaft with the motor using a coupling





Wheels: *Steering & Suspensions*

There are a number of strategies for steering a mobile robot. Each of which has unique kinematic and dynamic properties. The easier the implementation method, the more modest the performance:

- *Differentially drive the back wheels while the front wheels are on fixed axles:* This requires good control of the differential speed of the rear wheels, and the front wheels must be able to skid across the ground. Thus the rear wheels should have a high friction surface (e.g., rubber or sandpaper glued to the wheel surface) while the front wheels should have a low friction surface (e.g., plastic).
- *Differentially drive the back wheels while the front wheels are mounted on casters:* This requires good control of the differential speed of the rear wheels.
- *Use 4WD:* The wheels on each side (or use treads) of the vehicle are torsionally connected together, and skid steering (differential speed between the two sides of the vehicle) is used to turn the vehicle.
- *Drive the rear wheels with a differential and steer the front wheels with a linkage:* This uses one actuator for steering and thus reduces the total tractive effort of the vehicle. If the contest requires more maneuverability than tractive effort or speed, this may be the most desirable strategy. One must make sure to check the required and achievable tractive effort!
- *Drive the rear wheels with a differential and steer a single front wheel using a gear that is driven by a pinion attached to a motor:* This is the most maneuverable system, and the 3-point support of the vehicle eliminates the potential need for a suspension, but the vehicle may be less stable.

For each of these strategies, there may be many different concepts with which to achieve the strategy. As always, cost verses performance must drive the design decision. In general, the best all-around performance system is the rear wheel drive with castored front wheels. The best brute-force machine with good maneuverability is typically the 4WD or tracked vehicle with skid-steering. The most maneuverable, and strangely not often seen, is the three wheeled vehicle where the two rear wheels are driven with a differential and the vehicle is steered using a single front wheel attached to a rotating platform that is steered by a motor.

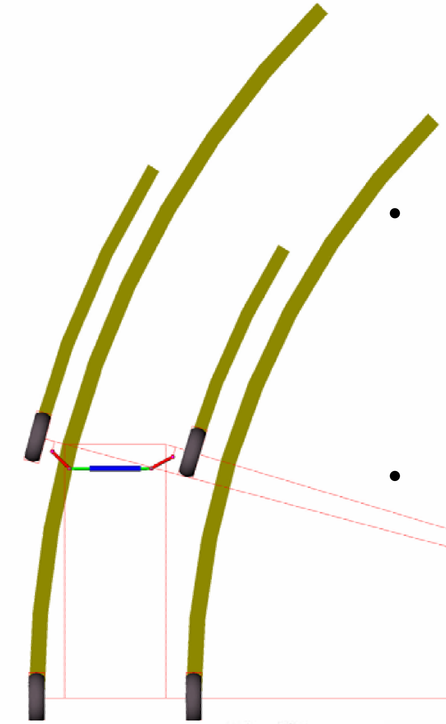
The vehicle suspension also greatly affects controllability and performance. Most robots built for contests do not have suspension elements, and instead rely on the compliance of the support structure and prayer to ensure that their wheels always contact the ground. When shock loads are to be incurred by driving fast over rough terrain, however, independent suspension is a must to maintain contact with the ground and to maintain control. In addition, a suspension can soften impacts and help prevent the breaking of components. Accordingly, there are various forms of suspensions. Once gain, the cost increases with the performance:

- No suspension and just rely on the compliance of the support structure
- Use a pivot shaft to support one set of wheels so they can move out of plane with the rear wheels (like the front axle of a small tractor)
- Independent suspension for at least two if not four of the wheels

In order to determine which suspension is best, consider the functional requirements for your overall strategy, and assess your available time. In general, for most robot contests, unless the contest requires a great deal of speed over a rough terrain, a pivot shaft suspension is more than adequate. Alternatively, a three wheeled robot contest vehicle probably does not require a suspension.

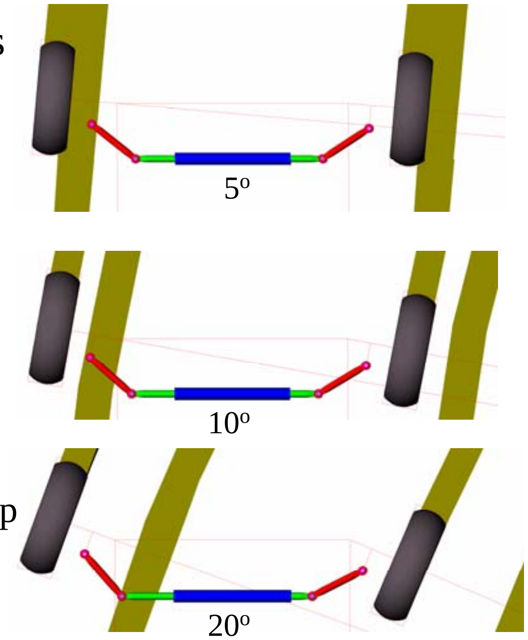
Assess the planarity of the contest tables and judge the risk that your vehicle may end up being supported by three wheels only. What countermeasures can you implement to ensure that, no matter what, the driven wheels always remain in contact with the table? Does this adversely affect the stability of your vehicle? Would you be better off adding a suspension? What is the simplest suspension that you require? If you were to make a 3-wheeled vehicle, what countermeasures might your propose to minimize the risk of tipping? Would outrigger skids or wheels be effective? How can you easily mount the third wheel and attach a gear to it so it can be steered? Can you attach a gear to a castored wheel? Where should the steered wheel be located, in the front or the rear of the vehicle?

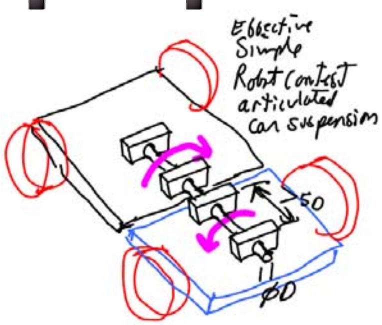
Wheels: *Steering & Suspensions*



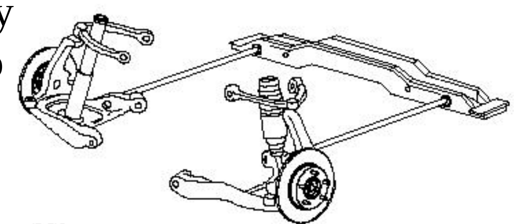
- A *steering linkage* ensures that all the vehicle's wheels axes of rotation meet at a single point, which is the instant center (center of the turning radius)
  - The Ackerman linkage is a 6-bar linkage developed for automotive steering
- A *suspension linkage* ensures that all wheels keep contact with uneven ground
  - The linkage allows vertical motion of the wheel, but maintains wheel alignment
    - Advanced suspensions may cant (tilt) wheels to help with high speed cornering
  - Recall page 3-26 and the concept of elastically averaged design
- Vehicles with crawler tracks (e.g., bulldozers) typically allow at least one of the Crawler tracks to pivot to help ensure the treads to maintain ground contact

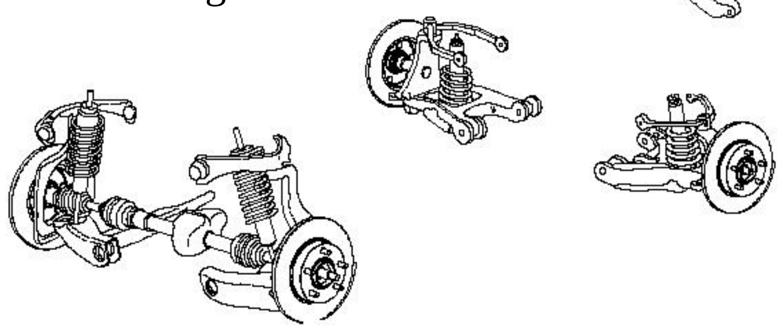
Instant center





http://www.cannondale.com/bikes/02/cusa/





Special thanks for the truck front suspension images to F. Zafar Shaikh, Senior Technical Specialist, Ford Motor Company

Wheels: *Steering & Suspensions*

Have you ever taken the time to carefully examine the steering and suspension on a real car? It can be difficult to pick it up and turn it upside down (physically, but mentally its a great exercise!). Have you ever carefully examined a radio controlled model car? High performance radio controlled cars are amazing little machines with full suspensions, differentials, and steering mechanisms. Their critical performance features are often scaled models of full-sized high performance machines. What to scale and what to redesign for is governed by consideration of:

- Kinematics: motion, accuracy, space
- Dynamics: forces, speeds, life
- Economics: design, build, maintain

For example, the pictures show close up photographs of a Traxxas RC 4WD off-road truck. The left side sequence shows the front suspension in the relaxed, half-loaded, and fully loaded positions. Sketch the linkage and label the critical elements. Notice that the suspension links themselves are essentially a 4-bar linkage. The shock absorber and suspension spring are colinear, and are anchored to the middle of the bottom link. Would this be done in a full-scale truck? Why or why not? Do all things automatically scale from large to small systems? The answer is “no”; however, we can always learn from one scale as we design on another!

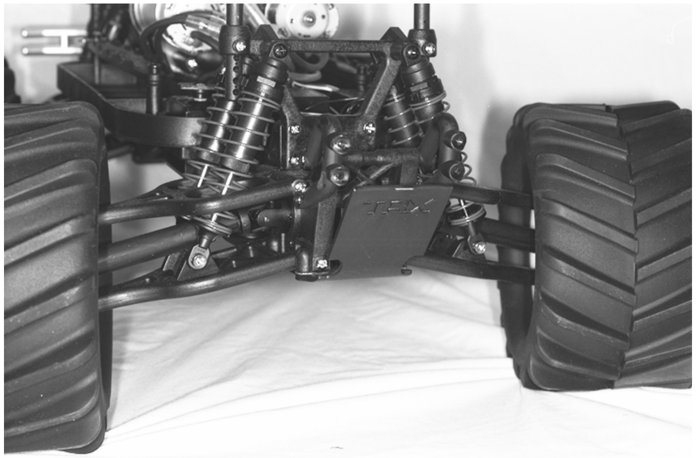
Consider nature, where a cat can jump off a table that is five times its height. What would happen if an elephant jumped off a 20 meter high ledge? Often, the smaller something is, the stronger it is in proportion to its size., not just by having less chance of defects initiating a failure, but from a mass/strength perspective. In the case of the model truck, the spring/shock absorber can be more conveniently attached to the middle of the lower link. In a full size truck, it would be more likely attached to near the wheel. It is a combination of the kinematics, dynamics, and economics that drives this difference between the full-size and scaled trucks.

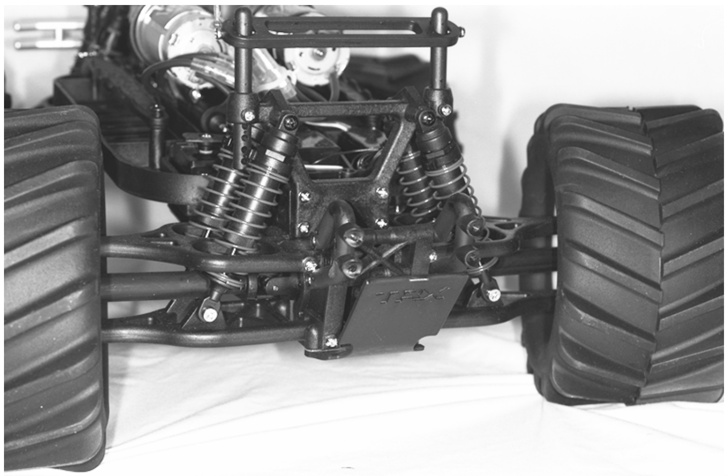
The suspension allows the body to move with respect to the wheels, so the driveshaft must be able to move up and down, and also accommodate the turning of the wheels. How many degrees of freedom are required from this joint? Does the driveshaft change length? How does it also accommodate misalignment? Look at the drive shafts emanating from the black box in the center of the underside of the truck. Why does there seem to be a tube within a tube? What kind of joint would allow the smaller tube to slide in and out of the larger tube while still transmitting torque? Why is the author asking so many questions instead of just providing a figure with lots of labeled items?

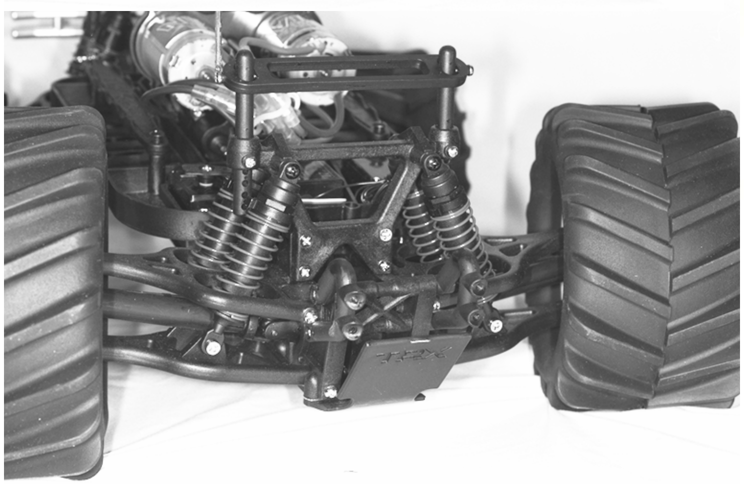
Now look at the steering linkage shown in the figures on the right side of the page. Sketch the primary elements. How does the link that connects the actuator which creates the force to cause the wheel to turn, connect to the system? How does the link that connects to the wheels themselves accommodate the up and down motion of the suspension? How many degrees of freedom are required at each joint? Why does it seem that there is a rotary motion link attached to a four bar linkage that then pushes links attached to the wheels? Would a full-size truck or car use such a system? Why can this system get away with this type of design? Does it have anything to do with the mass of the truck relative to its size?

In addition to shafts that must accommodate large motions, there are many precision shafts that support gears which must remain in precise alignment. These shafts must be supported by precision bearings (see Chapter 10!) which get hot when they spin at high speed. What about thermal expansion? Even very small displacements can cause HUGE forces, which readily destroy bearings when they are applied to very stiff systems. Hence the importance of *deterministic design* and the identification of both the *degrees of freedom* required and the *sensitive directions* of the system!

All those fundamentals of Chapter 3 apply over and over! Wire the fundamentals into your bio neural net so whenever you are faced with a problem (opportunity!), immediately dissect it in terms of the fundamentals! Always ask “why” (because if you understand the fundamentals, you can solve anything, and it makes getting a date a lot easier). This is why is so much FUN to do the MENTAL exercise of taking things apart and analyzing them. As you do these exercises, you train your brain, and you will more readily notice details and impress your friends (and make a better living!).

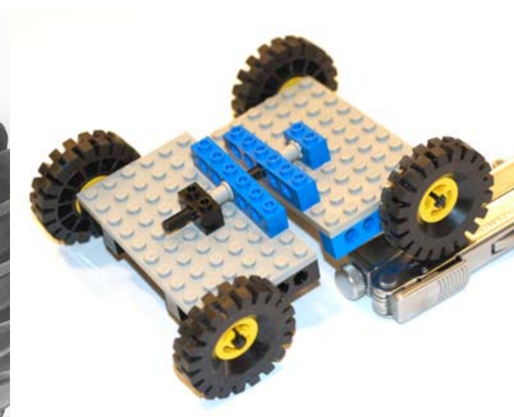


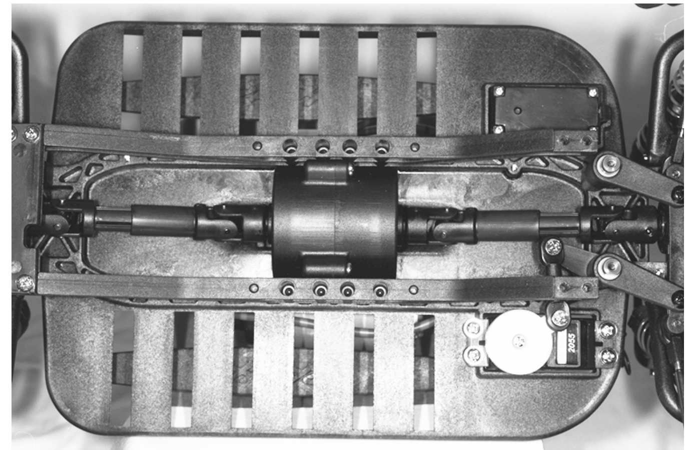


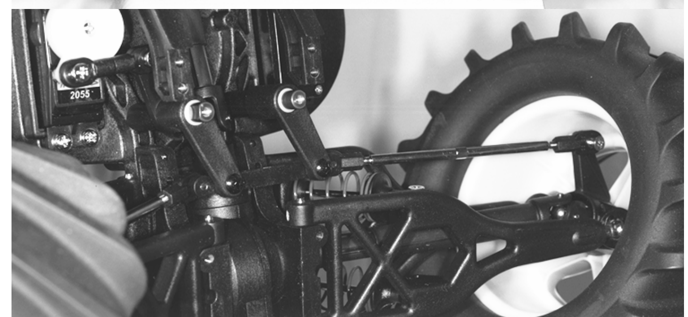


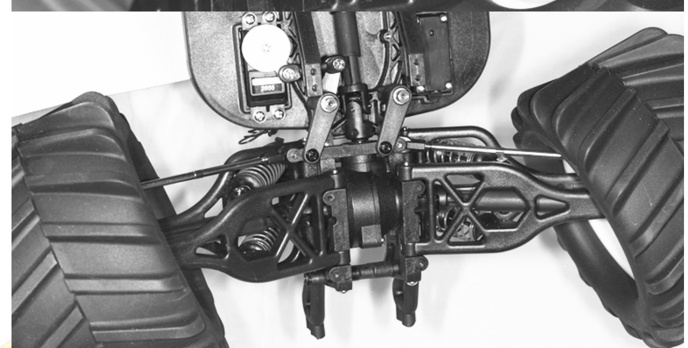
Wheels: *Steering & Suspensions*

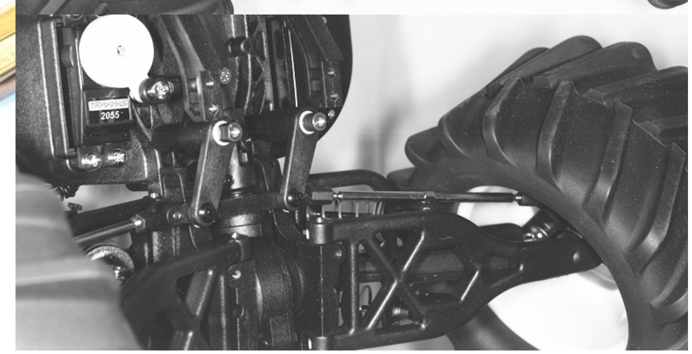
- How to transmit power to the wheels **<u>&</u>** enable them to move with respect to the vehicle to absorb shocks, move across uneven surfaces?
- How to transmit power to the wheels and enable them to steer?
- How does a tractor's suspension work ?
  - Do <u>you</u> really need 4 wheel independent suspension?











Wheels: *Clutches & Differentials*[1]

Clutches and differentials are very important transmission components whose operation is normally transparent to the user until they are needed. A clutch normally behaves as a rigid element until a certain torque is exceeded, and then it slips. A standard or "open" differential equalizes the torque on two output shafts while constraining the sum of their rotary displacements to be equal to the input rotary displacement.

In the case of an automotive clutch, the clutch allows slip between the engine and the transmission so that the engine will not stall at low speeds. In the case of a robot or manufacturing equipment, a clutch is typically used to prevent overloading of components. For example, a clutch can prevent gear teeth from being sheared off when a driven member encounters a rigid object. Clutches can also be configured to resist torque in only one direction. A simple form of this type of clutch can be made by pressing a spring over the ends of two shafts, and anchoring it only to the driving shaft. For torque applied in one direction, the spring winds tighter and transmits torque. For torque applied in the opposite direction, the spring unwinds and torque is not transmitted. The kinematics of a clutch are transparent when it is engaged, transmitting torque without slip, and it functions as a revolute joint when it is disengaged. The dynamics of a clutch are more complex, because they depend on the friction coefficient; however, because of the requirement for high friction to enable the clutch to transmit high torque, they act as either rigid elements or very highly damped elements.

The most common form of clutch is a simple mechanical clutch where one plate is forced against a second plate through the use of a spring. By adjusting a screw to change the pressure on the spring, the slip torque can be adjusted. The design parameters affecting a clutch's torque capacity are: the diameter of the plates, the coefficient of friction, and the force between the plates, A subtle design parameter, however, is the means by which the clutch plate is mounted that enables it to move forward and engage its mate. If it slides on a spline shaft, then there will be backlash (see page 3-23), and the clutch is not suitable for use in devices where a high degree of precision is required in either direction of motion. For precision applications, the clutch plate should be mounted on a diaphragm.

1. Special thanks to Greg bean for his most useful edits and comments!

The force applied to the clutch plate can be applied mechanically or electromagnetically, and the clutch can either be normally engaged or normally disengaged. In the former case, springs maintain pressure between the elements, and a linkage or electromagnetic field is used to release the pressure.

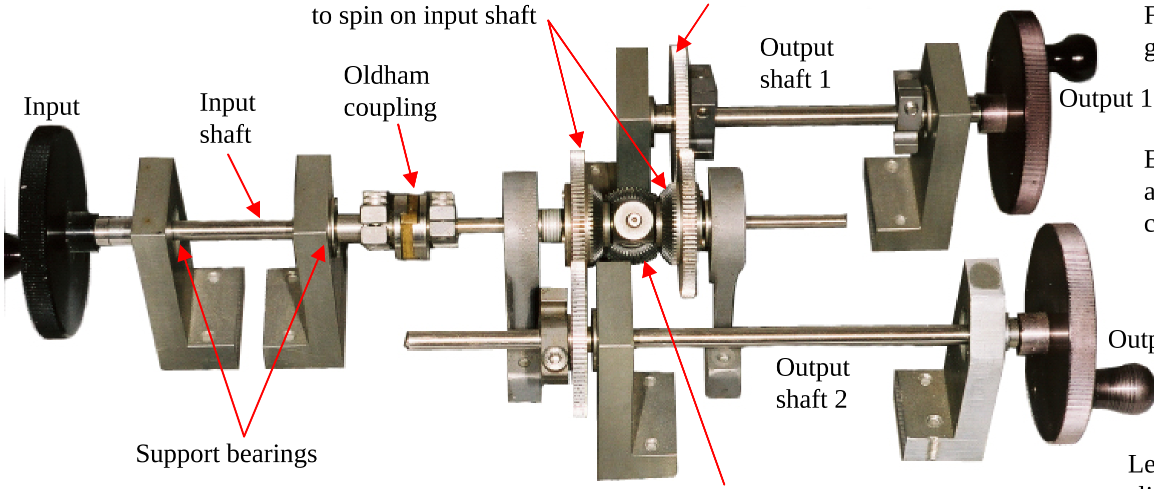
Once the clutch engages and power makes its way to the driveshaft, a differential is used to divide the power between the wheels. A differential has a single input shaft and two output shafts, and it has two functions. The first function is to divide the input torque equally between two output shafts, so each shaft in effect sees a torque source and can thus rotate at a different rate than the other if needed. The classic use is in the rear end of a car which allows the outside wheel to go faster than the inside wheel when the car is going around a curve. However, when one wheel is on ice and the other wheel is on pavement, the differential can only apply to the wheel on pavement the same torque that is applied to the wheel on ice, not very much, since the wheel on ice easily slips. The vehicle may thus get stuck, with the total torque on the output shafts insufficient to move the vehicle. The second function is to act as the final gear ratio between the input shaft and the output shafts. Differential gear ratios of up to 4:1 are not uncommon.

Why not just drive one wheel and avoid the need for a differential? If only one wheel in a vehicle were driven, then the maximum tractive force would be less than half that if two wheels were driven with a differential. Only driving one of the wheels would also cause the vehicle path to curve, because the driving force is not applied through the vehicle's center of friction. For a robot contest vehicle, if a single motor has all the power that is required to move the vehicle, and the mass distribution is such that the full torque of the motor can be applied to the driven wheel without causing it to slip, then a differential is not needed. If the wheel is likely to slip, then a differential can be used to enable the single motor to drive two wheels. Of course, it also depends on what type of steering strategy you are using.

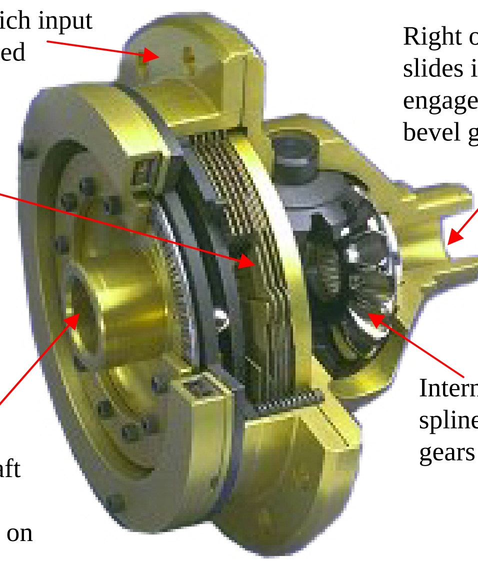
Can you make a clutch from elements in your kit to prevent critical gears from being damaged? Is it safe to press a gear onto a shaft and rely on the interference-fit to act as a clutch? What design parameters affect the interference-fit force? Can you machine flexural elements into the gear's hub so as to ensure proper preload between the gear and the hub to maintain a desired slip torque? Can you make a differential with the gears you have, or do you need to use a commercial differential?

Wheels: *Clutches & Differentials*

- Clutches and differentials allow for differential velocities between rotating shafts
  - A classic example is when a car makes a turn, all the wheels have the same instant center, but the radius from each wheel to the instant center is different
    - Thus each wheel travels a different distance along an arc in the same time, and a *differential* is needed to enable the speed difference to occur
      - However, if one wheel is on ice, and another wheel is on dry pavement, the wheel on ice will spin
    - Limited slip differentials typically use spring-loaded clutch plates to ensure some torque is always transmitted to both wheels
    - Other designs use a centrifugal force to engage locking features
    - New designs use electromagnetic clutches to optimize torque transfer to the wheels
  - Clutches can be activated mechanically or electrically to control the torque transmitted between shafts, while allowing differential velocity to occur (see page 6-17)



Electric clutch allows limited slip across the differential, or the differential to lock forcing both shafts to be driven. See: http://automotive.eaton.com/product/traction_stability/intellitraxx.html



See page 6-19 for a differential a student built for his robot

Cams[1]

A *cam* is a machine element whose specially shaped *lobes* are followed by a *cam follower,* which causes the cam profile to be imparted on another object. A common application is in an internal combustion engine where the cam is driven by a chain or belt connected to the *crankshaft*. The *cam lobes* are thus synchronized with the crankshaft rotation to open and close intake and exhaust valves as required. The shape of the cam *lobe* determines not only when a valve is opened, but how fast, and how long it stays open, which is called the *dwell time*. In a modern overhead cam engine, the *cam* pushes on a *cam follower,* which pushes on the *valve stem*. The *valve* is held in a normally closed position by *valve springs*, which, through the chain of elements cause the cam follower to remain in contact with the cam and keep the valve normally closed. This may seem like an archaic system ripe for replacement with modern electromagnetic actuators; however, for high reliability, low cost, and compact size in a hostile environment, cams still rule.[2]

When the cam profile exists on the surface of a cylinder, it is called a *barrel cam*, and two degrees of freedom can be defined. Barrel cams control *screw machines*, which are multi spindle lathes that make small round parts. Each spindle holds the same type part between stations. At each station, a dedicated tool performs one operation on the part. Barrel cams typically have several tracks for controlling several tool motions. Screw machines make all the zillions of small round parts of which some are used in virtually everything. In fact, one of the common products made cheaply on screw machines are screws! Most new screw machines are computer controlled, but countless old ones keep on camming!

Another example of a barrel cam is the Yankee™ screwdriver which converts downward motion of a handle held by the user into rotary motion of a screwdriver tip. In a robot for a design contest, for example, a leadscrew nut may be prevented from rotating by trunnions (ears) that stick out its side and slide in a axial grooves in a follow barrel cam (the nut moves inside the cam). As the screw shaft rotates, the nut will translate until the axial groove transitions to a circumferential groove and instead of translating, the nut now rotates. The system can be made to translate and rotate (move and dump) using a single rotary motor and a barrel cam.

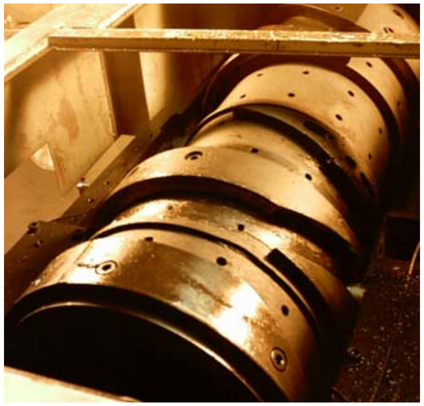
Cams can also be used in situations where they are stationary and the follower moves along their contour. An early example, which helped give birth to precision machine tools which in turn made our modern world possible, is the *corrector cam*. To obtain linear motion, machine tools use leadscrews which convert rotary to linear motion, as discussed in detail in Chapter 6. The lead on the screw, can only be made so accurate. A corrector cam placed alongside the leadscrew with a cam follower connected to an arm projecting out from the nut will cause the nut to rotate forward or backward relative to the leadscrew in order to compensate for small lead errors. Today, repeatable errors are mapped and the error correction is done digitally.

There are many types of cam followers. *Radial followers* are used with the heaviest loads to convert rotary motion into linear motion, but the range of motion of the follower is modest. *Offset followers* enable greater motion of the follower, but require greater torque. *Swinging followers* convert cam rotation to oscillating rotary motion. A roller tip minimizes friction, but the roller has less load capacity than a flat follower. A compromise is the spherical follower. All these followers, however, are limited in how rapidly the lobe can change shape. When extremes in lobe shape occur, a knife edge follower may be used, but the load capacity will be more limited. Good lubrication is key, and Hertz contact stresses must be controlled (see Chapter 9).

Designing a cam is a straightforward inverse problem. Given the desired output motion as a function of input angle, the radius of the lobe at the point of contact between the cam follower and the lobe can be determined. One must not only evaluate the kinematics of motion, but also the dynamics of motion including accelerations of the follower to determine the spring force necessary to ensure that it remains in contact with the cam. If the spring force is too high, the system will wear out too soon, and if it is too low, the follower will lose contact with the cam surface. Fortunately, just as with linkage design programs, there are a plethora of cam design programs, many of them available for free on the internet!

Where might a cam enable your robot to achieve non-simple motion, but without the space requirements of a linkage? Can a barrel cam be used to enable one actuator to perform a complex motion?

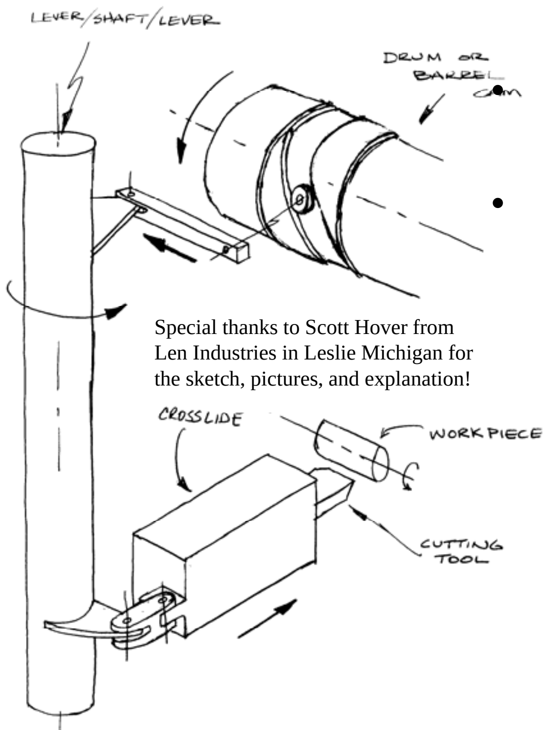
1. Cams might have been discussed in the context of linkages, but since they are often wheel-like and directly connected to power sources, it is equally valid to discuss them here.
2. M. Fischetti, “Why Not a 40-MPG SUV”, Technology Review, November, 2002 PP 40-46



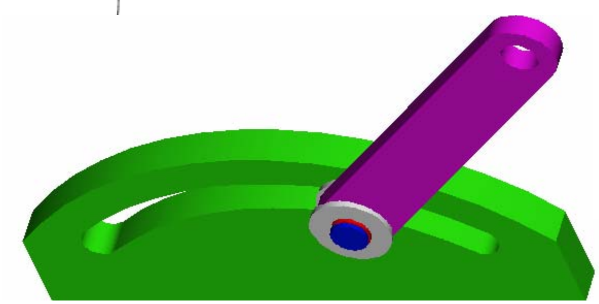
Cams

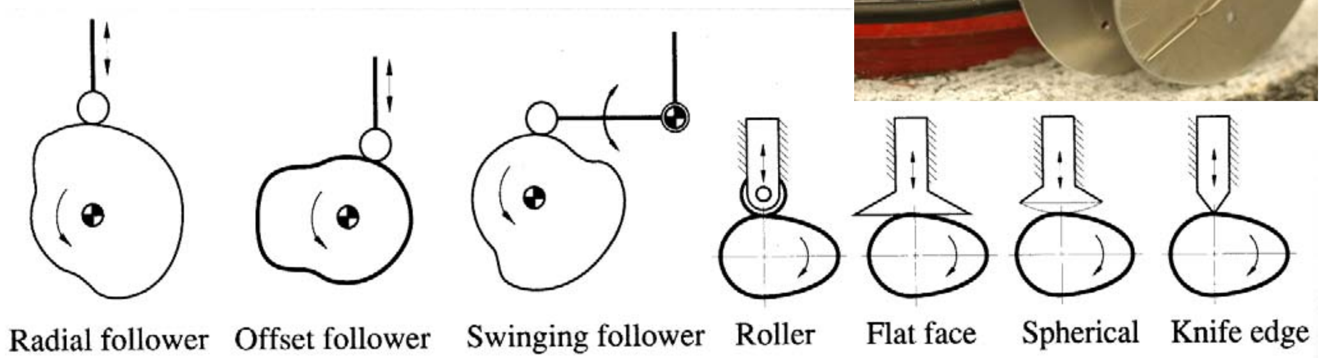
Stanley Tool Works' Yankee™ screwdriver

- A *cam* is a rotating shape whose angular motion is converted into output motion by a *cam follower* which rides on the cam surface
  - Rolling elements provide the highest degree of efficiency, but they take up more space
  - Sliding elements are very compact, and can be efficient of the speed is high enough to maintain oil film lubrication
- A *cam follower* is a rolling or sliding contact element that follows the contour of a surface and transmits the motion to a mechanism
- A *cam profile* can be designed to create corresponding *acceleration, velocity*, *position*, and *dwell* profiles in a mechanism (e.g., an engine valve)
  - Cam design software can create virtually any type of cam and provide all required engineering data
  - See for example http://www.camnetics.com/



Special thanks to Scott Hover from Len Industries in Leslie Michigan for the sketch, pictures, and explanation!





Shafts

Shafts may seem like relatively benign machine elements, but they are often subject to large cyclic bending and torsional loads. There are also many different components which might be attached to them in a myriad of different ways. In addition, to reduce bearing friction losses, it is desirable to minimize shaft diameter. All these facts combine to make shaft design one of the more challenging aspects of machine design. The first step in shaft design is to assess the bending and torsional loads on the shaft. The bending loads can be determined from a free-body diagram of the proposed system, taking care to note where the shaft is supported in bearings and weather the support points act as simply supported constraints, or moment supporting constraints. A bending moment diagram can then be made to predict the bending moment as a function of position along the shaft. The torsional load Γ (N-m) can either be known by the machine specifications directly or determined from the power P (Watts) and speed ω (radians/second):

$$\Gamma = \frac{P}{\omega}$$

Given an understanding of the bending and torsional loads on a shaft, the next step is to calculate the stresses and deflections in the shaft, especially given the presence of large stress concentration factors that will exist wherever components are attached to the shaft. In addition, wherever the shaft receives moment support from a set of bearings, the interface between the shaft and the bearing acts as a discontinuity and the appropriate stress concentration factor should be applied. The basic formulas for determining bending, torsional, and shear (from radial loads) stresses in shafts are well known:

- Bending: $\sigma = Mc/I$ where $I/c = \pi D^3/32$
- Torsion: $\tau = \Gamma r/I$ where $I/r = \pi D^3/16$; $\phi = \Gamma L/GI$ where $I = \pi D^4/32$
- Shear: $\tau = F/A$ where $A = \pi D^2/4$

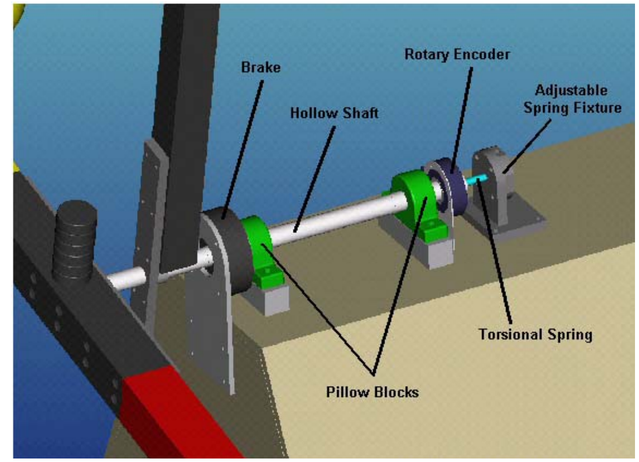
Because shafts typically rotate, they can experience very high cyclic loading, and thus they are especially prone to fatigue. In general, engineers are taught in their basic mechanics course that steel alloys have an infinite fatigue life if the stress is kept below one-half of the yield stress. However, this is just a rule of thumb for initial layout of a design. When it is time to select the actual alloy and its heat treatment, one must be very careful to check the endurance limit specifications for the chosen material. Different alloys behave differently at the upper limits of their strengths. Furthermore, what may seem like a properly designed shaft may fail in fatigue due to an overlooked stress concentration, which is why the next three pages are devoted to highlighting some of the more common loading and geometry configurations. Typically a design engineer will use a spreadsheet or program they create to do a parametric study of how the shaft stress will vary as a function of varying geometric parameters and loads, and then check the final design using finite element analysis.

Its not just the stress that is important in shaft design, deflections of the shaft can cause bearings to overload or seals to fail. If a bearing is designed to primarily withstand radial loads, and the shaft which it supports is too long, too large a moment may be created. Unless the bearings can accommodate the resulting angular deflection, the bearing may fail prematurely. Roller and needle roller bearings are particularly sensitive to angular deflections. If a sliding contact bearing (bushing) is used, clearance must exist between the shaft and the bushing, excessive bending of the shaft may cause the clearance to close causing the bearing to seize.

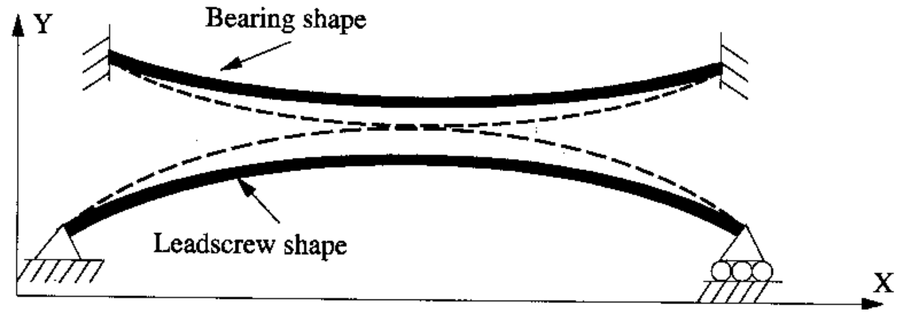
When shafts are coupled to other elements, such as a leadscrew driving a nut attached to a carriage, there can never be perfect alignment between the shaft and the other element. As the system moves, there has to be sufficient compliance somewhere in the structural loop to enable the misalignment to be accommodated without overloading the system. Typically the misalignment is accommodated either by clearance between members, compliance in the shaft, or with a flexible coupling. In all cases, however, one must determine the accuracy and repeatability requirements of the system, and make sure the method used provides adequate performance.

Shaft alignment in design contest robots is one of the most overlooked aspects of their design. Carefully review your design and see how features can be machined in place or made easily adjustable so that shafts can be easily aligned. The next biggest problem area is shafts that deflect too much and cause components to bind. Do a careful review of shaft loading and component clearances to guard against this failure mode.

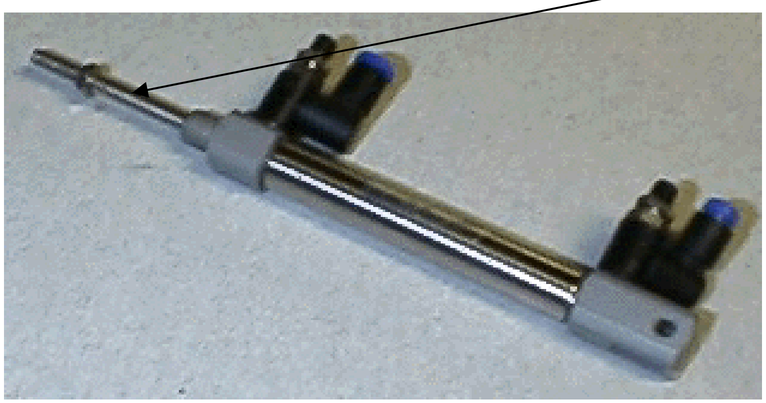
Shafts



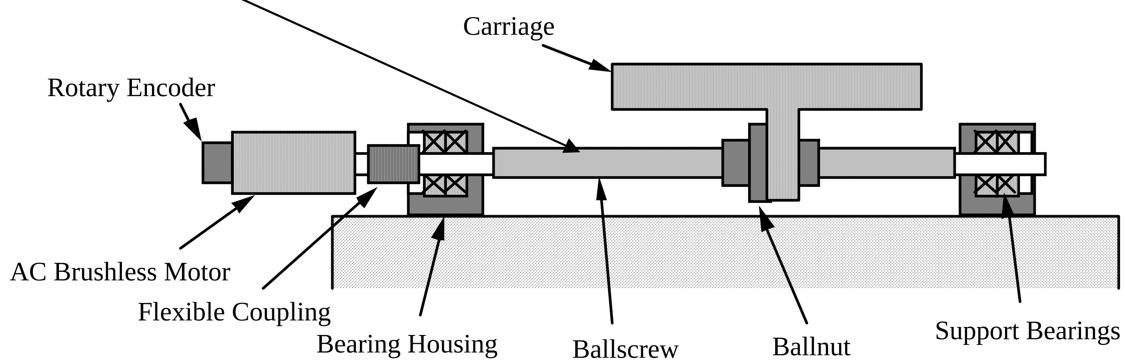
- Shafts transmit rotary and linear power via motors, leadscrews, pistons…
  - Shafts are one of the most common machine elements
- The primary design issues are:
  - Loading conditions and stress concentrations
  - Mounting
  - Component Attachment



Shafts



pneumatic or hydraulic cylinder



Leadscrew driven carriage

Shafts: *Axial Loading*

Axial loading of shafts may be tensile or compressive, and the stress concentration factors shown in the table apply in either case. When loads are compressive, one must also make sure that the shaft does not suffer from a buckling instability, as discussed on page 6-6. Pneumatic and hydraulic cylinders' shafts often are subject to very high axial loads, and in addition, they often have end attachments that can be subject to large stress concentrations. Their shafts typically undergo higher compressive than axial loads, but they may also be subject to substantial tensile loads when double acting cylinders are used. In this case, since the loads alternate between tension and compression, fatigue cracks are more likely to grow than if the system were just subject to compressive loads.

Leadscrew shafts also typically experience very high alternating axial tension and compression loads. Leadscrews are also often placed in tension so that as the nut travels and heats up the shaft, thermal strains are replace the tensile preload strain and the lead (mm/rev) remains constant, as discussed in Chapter 6. Leadscrews usually have very simple end connections, so the primary place for fatigue is in the root of the threads. The stress concentration factor can be estimated using the third case illustrated in the table where the thread root diameter would be *r*.

Case 1 shows a typical situation where a shoulder in a shaft transfers axial loads from the shaft to the inner ring of a ball bearing. The inner ring of the ball bearing will be chamfered, which allows for a small radius to relieve the stress concentration. Note if the radius is not made correctly, e.g., it is made too big, then the bearing would not seat against the shoulder and the design would fail. For greater stress relief or more manufacturing robustness, *Case 2* shows how the shaft itself can be grooved to yield in effect a sharp corner against which the bearing inner ring can seat., However, if the groove is made too large, it reduces the shaft diameter and the stress is too large. In deciding which design to use, the designer can use the equations for both cases to select the best design for their application. The spreadsheet *Shaft_axial.xls* compares these two cases. Sample output is shown below. Within the spreadsheet you can also parametrically explore the trends that occur as the corner or undercut radius is varied. When is it better to use a radiused corner and when is it better to use an undercut?

shaft_axial.xls **Radius transition or undercut between diameters?**	
Large diameter, D	**14**
Small diameter, d	**8**
Axial load (N)	**20**
Major shaft diameter (mm)	**14**
transition radius (mm)	**0.5**
Stress concentration factor fillet shaft, Kt	**2.39**
Stress concentration factor undercut shaft, Kt	**2.14**
Shaft length (mm)	**50**
Shaft modulus, E (GPa)	**2.00E+11**
Area at fillet shaft diameter (mm^2)	**50**
Area at fillet shaft diameter (mm^2)	**38**
Fillet design stress, sigma (MPa)	**0.950**
Undercut design stress, sigma (MPa)	**1.111**
Shaft deflection along minor diameter (micrometers)	**0.099**

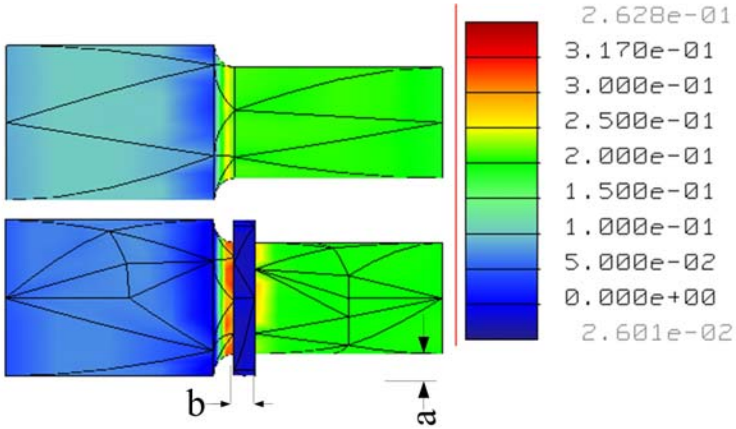
Case 3 and *Case 4* show two alternate low cost methods of affixing a component to a shaft using a pin through the pulley hub and the shaft, or a key in a keyway cut into the shaft and the pulley inside radius respectively. The former is fast and simple, but results in large stress concentrations. As can be seen on the following pages, this greatly reduces the torsional and bending strength of the shaft. A pin through a shaft should only be used for modest loads or where there are no bending or torsional loads from the hole onward. This would be the case for a cantilevered shaft with a pulley or a wheel on its end: the pin should go on the outside towards the end of the cantilevered shaft.

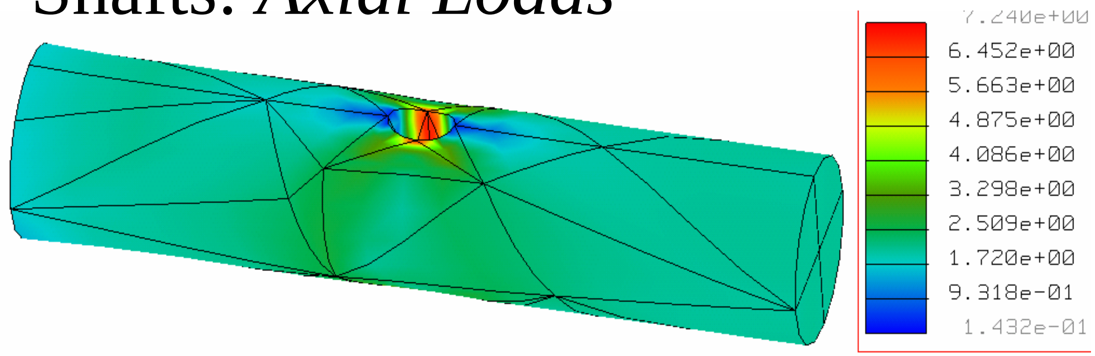
A variation on the radiused corner uses a narrow straight mini-flange near the base of the radiused corner. This small section is large enough to provide a square shoulder against which a component can be axially located. The comparison table shows it is too small to create a large stress concentration.

What shafts in your design might be overstressed or become fatigued? What are the stress concentration implications for bolt design? Are you using a long small diameter leadscrew (e.g., a piece of threaded rod) that may buckle under high leads? Do you have linkages with thin round shafts as members?

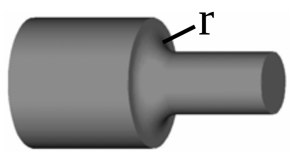
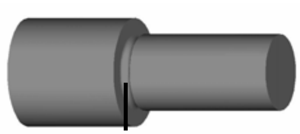
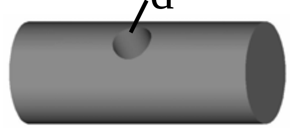
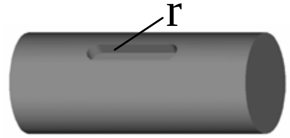
Shafts: *Axial Loads*

See *Shaft_axial.xls*





	$K_{t\ radius}/K_{t\ mini\ flange}$		
b/a	axial	torsion	bending
0	1.0	1.0	1.0
0.5	1.2	1.4	1.1
1	1.3	1.4	1.1

	Shaft loaded by axial force F	Stress	Stress concentration factor
Case 1	D  d r	$\sigma = \frac{K_t 4F}{\pi d^2}$	$K_t = 1 + \frac{(r/d)^{-0.36-0.2(D/d)}}{5+0.12/(D/d-1)}$
Case 2	D  d r	$\sigma = \frac{K_t 4F}{\pi (d-2r)^2}$	$K_t = 1 + \frac{(r/d)^{-0.511-0.34(d/(d-2r)-1)}}{3+0.507/(d/d-2r-1)^{0.42}}$
Case 3	D  d	$\sigma \approx \frac{K_t F}{0.25\pi D^2 - dD}$	$K_t \approx 1+0.65/(d/D)^{0.275}$
Case 4	D  r Corner r Width b Depth t	$\sigma \approx \frac{K_t F}{0.25\pi D^2 - bt}$	$K_t \approx 1.5$ B=D/4, t=D/8

Shafts: *Torsional Loading*

Torsional loading of shafts usually results in the highest number of cycles experienced by just about any machine element, except the individual balls in ball bearings. In addition, thin long shafts can also suffer from buckling instability, but this rarely occurs. Torque transmitting shafts are commonly found in motors, transmissions, and machine tool spindles. In addition, leadscrew shafts and bolts (screws) also are subject to large torques. Complicating matters are the various machine elements, such as gears, pulleys, and attached loads, which must be attached with sufficiently robust means to transmit the torque; however, the features used to transmit the torque, such as pin holes and keyways, can significantly weaken the shaft.

shaft_torsion.xls **Radius transition or undercut between diameters?**		
Large diameter, D	**14**	
Small diameter, d	**8**	
Motor Power (W)	**20**	
Motor speed (rpm, rad/sec)	**100**	**10.47**
Shaft torque (N-m)	**1.910**	
Major shaft diameter	**14**	
transition radius (mm)	**0.5**	
Fillet stress concentration factor, Kt	**1.45**	
Undercut stress concentration factor, Kt	**1.52**	
Shaft length (mm)	**50**	
Shaft modulus, E (GPa)	**2.00E+11**	
Shaft Poisson ratio	**0.29**	
Shear modulus, G (GPa)	**7.75E+10**	
Polar I at fillet shaft diameter (mm^4)	**402**	
Polar I at undercut shaft diameter (mm^4)	**236**	
Fillet design torsion stress (MPa)	**27.6**	
Undercut design torsion stress (MPa)	**43.0**	
Shaft twist along minor diameter (deg, rad)	**0.18**	**3.06E-03**

Whenever a machine element is attached to a torque transmitting shaft, at the attachment point, force must “flow” from the shaft and into the element so it can be transmitted by the element (e.g., a gear). Thus if the interface between the shaft and the element is sharp and has a discontinuity, a stress concentration will arise. Given that elements attached to shafts also often have to be precisely axially aligned, it is common for the element to be registered against a shoulder. This gives rise to the mounting conditions of *Case 1* and *Case 2* as shown in the output table shown from the spreadsheet *Shaft_torsion.xls.*

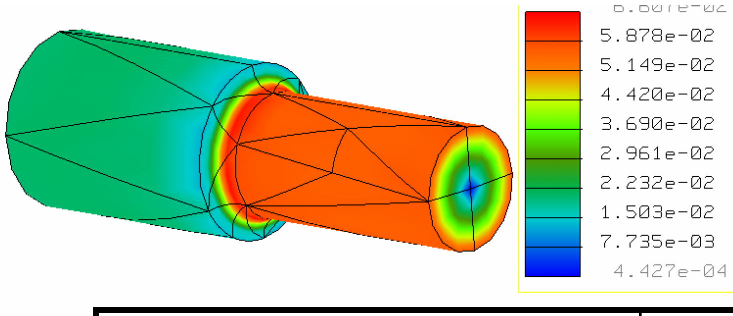
The bore through the machine element, such as a gear, will be chamfered, and this allows for a small radiused corner on the shaft to relieve the stress concentration. If the radius is too big, then the element will not seat against the shoulder and the design will fail. If a greater relief or more manufacturing robustness is required, Case 2 shows how the shaft itself can be grooved to provide a sharp corner against which the machine element can seat. However, if the groove is made too large, it reduces the shaft diameter and the stress is too large. In deciding which design to use, the designer can use the equations for the two cases to select the best design for their application. The spreadsheet *Shaft_torsion.xls* compares these two cases. Sample output is shown below, but within the spreadsheet you can also parametrically explore the trends that occur as the corner or undercut radius is varied. When is it better to use a radiused corner and when is it better to use an undercut?

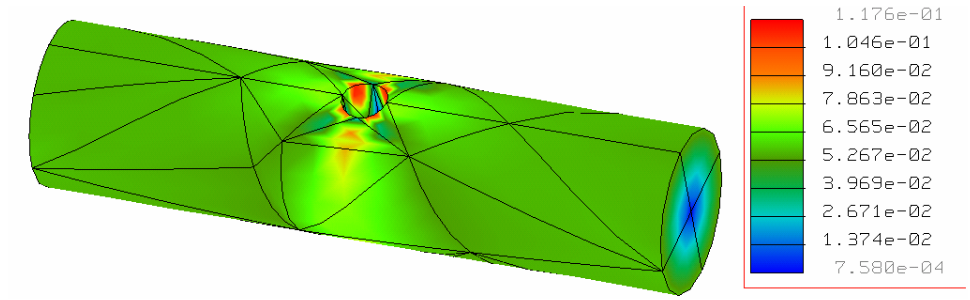
Once again, *Case 3* and *Case 4* show two alternate low cost methods of affixing a component, say a pulley, to a shaft using a pin through the pulley hub and the shaft, or a key in a keyway cut into the shaft and the pulley inside radius respectively. The former is fast and simple to do, but results in large stress concentrations which greatly reduces the torsional and bending strength of the shaft. Thus it should only be used for modest loads or where there are no torsional or bending loads from the hole onward. For the case of a cantilevered shaft with a pulley or a wheel on its end, the pin should go towards the end of the cantilevered shaft. This prevents bending loads from passing around the stress concentrating pin hole. The best, but most expensive, way to attach a torque transmitting element to a shaft is with a press or shrink fit, or a squeeze-type interface.

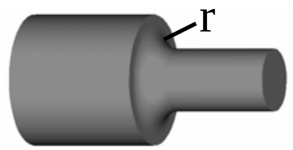
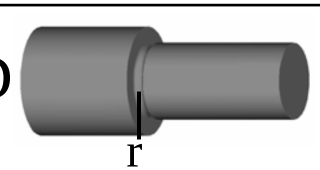
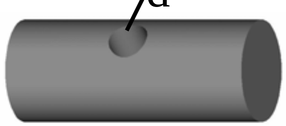
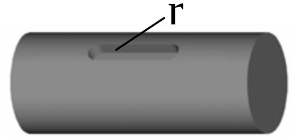
You undoubtedly have torque transmitting shafts to which you intend to attach torque transmitting elements such as pulleys or wheels. Run the spreadsheet and calculate the shear stress in the shaft. More than one student has been shocked to find that the torque output from a high reduction gearbox can shear their shaft. Many times the break occurs at a drilled hole for a pin!

See *Shaft_torsion.xls*

Shafts: *Torsional loads*





	Shaft loaded by Torque Γ	Stress	Stress concentration factor
Case 1	D  r d	$\tau = \frac{K_t 16\Gamma}{\pi d^3}$	$K_t = 1 + \frac{(r/d)^{-0.3-0.2(D/d)}}{13 + 0.3/(D/d-1)}$
Case 2	D  r d	$\tau = \frac{K_t 16\Gamma}{\pi\left(d-2r\right)^3}$	$K_t = 1 + \frac{(r/d)^{-0.609-0.146(d/(d-2r)-1)}}{5 + 3.73/(d/(d-2r)-1)^{0.252}}$
Case 3	D  d	$\tau = \frac{K_t \Gamma}{\pi D^3/16 - d D^2/6}$	$K_t = 1 + 1.47\left(d/D\right)^{-0.197}$
Case 4	D  r Corner r Width b Depth t	$\tau = \frac{K_t 16\Gamma}{\pi D^3}$	$K_t \approx 1.5$ B=D/4, t=D/8

Shafts: *Bending*

When wheels are attached to a shaft to support a vehicle, it is clear that there will be radial loads on the shaft that will cause bending stresses and deformations. In addition, when pulleys are attached to a shaft, it is clear that the cable forces will also radially load the shaft and cause bending stresses and deformations. Furthermore, when gears are attached to a shaft, there can be significant radial loads exerted by the teeth, and these radial loads also cause bending stresses and deformations. A quick rough estimate of the radial load on a shaft caused by a gear of pitch radius R_{pitch} (m) transmitting power P (Watts) and speed ω (rpm) is:

$$F_{radial} = \frac{30P}{\pi\omega R_{pitch}}$$

The stresses caused by radial loading induced bending moments can be significant, although resulting shaft deflections are generally in a non-sensitive direction with respect to maintaining proper center distance between gears. However, radial loads caused by gear teeth are also compounded by the pressure angle θ (degrees), or the angle at which the teeth engage, which tends to force the gear teeth apart. For example, for simple spur gears, this radial force, which is not significant with respect to stresses, can cause significant shaft deflection with respect to maintaining gear center distances, is:

$$F_{radial\_pressure\_angle\_force} = F_{radial}\sin\theta$$

As before, starting with *Case 1* and *Case 2* type loading, where a radiused corner or an undercut is used to register the axial position of a machine element such as a wheel, gear, or pulley, the stress concentration factor is greatly affected by the radius of the corner or the undercut. In both cases, increasing the radius decreases the stress concentration, but in the case of the radiused corner, it increases the likelihood that the machine element will not seat completely against the shoulder. In the case of an undercut, the larger the radius, the smaller the remaining diameter is directly beneath the undercut, and it is at this minimum diameter point that the stress needs to be determined. The spreadsheet *shaft_bending.xls* can be used to explore different shaft designs and trade-offs:

shaft_bending.xls **Radius transition or undercut between diameters?**	
Large diameter, D	**14**
Small diameter, d	**8**
Bending moment (N-m)	**10**
Major shaft diameter	**14**
transition radius (mm)	**0.5**
Fillet stress concentration factor, Kt	**2.89**
Undercut stress concentration factor, Kt	**1.99**
Inertia I at fillet shaft diameter (mm^4)	**201**
Inertia I at undercut shaft diameter (mm^4)	**118**
Fillet design bending stress (MPa)	**575**
Undercut design bending stress (MPa)	**592**

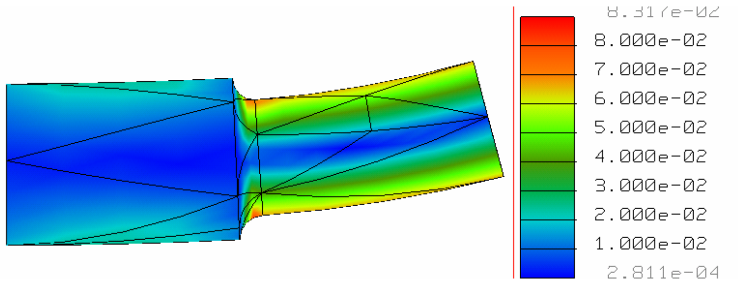
The effect of a hole in a shaft for pinning a component to the shaft, or of a key for enabling a shaft to transmit torque to a component once again also has an important effect both in terms of the stress concentration, and the fact that these features reduce the cross sectional area of the shaft.

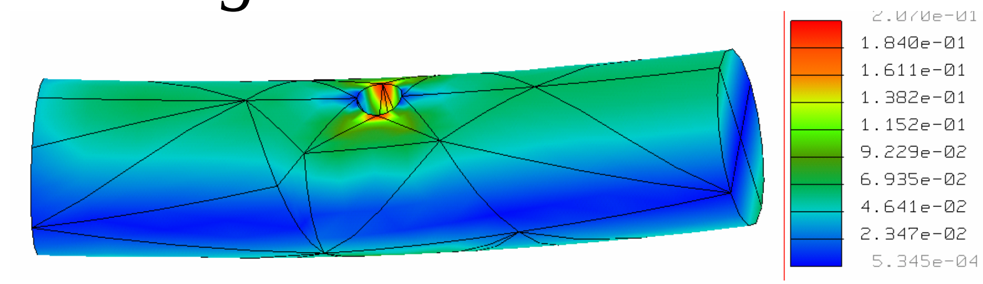
Since the bending loads may be the result of loads applied to the periphery of the machine element itself, the design engineer must also consider the effect of the loads on the component itself, and whether or not these loads are aligned radially, tangentially, or axially with respect to the shaft. Can these loads cause deformations in the element that is attached to the shaft that may weaken the attachment method? What is the level of shaft deformation, and can the resulting radial and slope deformations cause the shaft to overload bearings or reduce clearances to the point where the shaft rubs a bearing or a seal and causes it to fail? Can deflections cause a gap to open between a seal and a shaft and thereby cause the seal to leak?

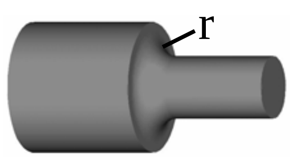
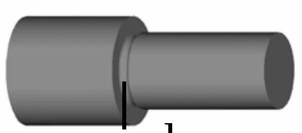
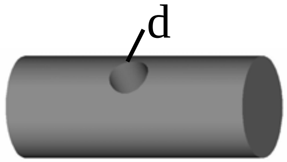
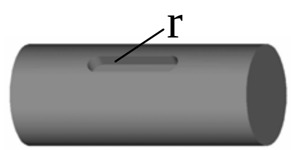
Many a student has designed a robot for a contest with gears that work fine when tested without significant loads. However, they sadly find that the teeth skip and strip under heavy loads. The cause is usually structure or shaft deflection caused by gear separation forces that increase the gear center distance too much. Check your shaft stresses and deflections! Check the stiffness of your supporting structure!

See *Shaft_bending.xls*

Shafts: *Bending Loads*





	Shaft loaded by Bending moment M	Stress	Stress concentration factor
Case 1	D  d	$\sigma = \frac{K_t 32M}{\pi d^3}$	$K_t = 1 + \frac{(r/d)^{-0.73-0.42(D/d-1)}}{5+4.38/(D/d-1)^{0.16}}$
Case 2	D 	$\sigma = \frac{K_t 32M}{\pi(d-2r)^3}$	$K_t = 1 + \frac{(r/d)^{-0.59-0.184(d/(d-2r)-1)}}{5+0.081/(d/(d-2r)-1)}$
Case 3	D 	$\sigma = \frac{K_t \Gamma}{\pi D^3/32 - d D^2/6}$	$K_t \approx 1 + 0.65/(d/D)^{0.275}$
Case 4	D  Corner r Width b Depth t	$\sigma = \frac{K_t 32M}{\pi D^3}$	$K_t = 1 + \frac{(r/d)^{-0.66}}{11.14}$ B=D/4, t=D/8

Shafts: *Mounting & Stability*

In addition to how components are mounted on shafts and the resulting stress concentrations that can occur, particular attention has to be paid to mounting a shaft to a structure. The three loading cases, axial, torsion, and bending, must be considered in conjunction with different possible connections between a shaft and a structure. Whether the shaft is fixed, or moving with respect to the structure makes a difference in selecting the mounting method.

Shafts that do not move with respect to a structure are commonly used as bearing rails on which linear bearings ride. They may also be fixed axles on which components rotate, such as pulleys or wheels with integral bearings. Fixed shafts can be held in place by a setscrew (bad), by clamping (squeezing) the shaft which is placed in a hole (good), interference-fit (good), or held in place in a hole with a threaded fastener (good). The method by which the shaft is anchored must also take into account the loads placed on the shaft.

If axial loads are to be heavy, then it might be a good idea to either pin the shaft in the hole, or to use a shaft with a shoulder so the axial load will be transferred by contact between the shoulder and the structure. Opposite the shoulder, the shaft would be threaded so a nut could resist tension in the shaft. If the shaft is to resist high torsion or bending loads, then its end can be pressed or shrunk into a hole, but then care must be taken to ensure that adequate interference is provided. An alternative method is to clamp the end of the shaft using a split housing as is shown in the figure for two bearing rails. Recall Saint-Venant, and seek to clamp the shaft by at least 3 shaft diameters if a true moment connection is to truly be established.

If the shaft is to be free to move with respect to a structure, then the bearings must be designed to accommodate bending deformations in the shaft without losing any desired bearing clearances. For example, whenever two shafts must operate in tandem either in the fixed or moving state, it is critical that their horizontal and vertical parallelism be made as good as possible so as to minimize bearing clearance that would otherwise be required to accommodate misalignment. When clearance exists in bearings, backlash can occur, and when backlash occurs in a carriage moving on bearing rails, it can have pitch, yaw, and roll errors, and these can be amplified by the Abbe effect to cause large errors in your machine! Alignment can best be achieved by line-boring mounting holes for the shafts, or their bearings, in the support plates at the same time. In other words, clamp the two plates that will form the supports together in the milling machine and bore the holes through both plates at the same time. Using common reference features on the plates and boring the holes at the same time ensures the holes will be as well-aligned as is possible.

For stability, the table shows formulas for determining static buckling loads and shaft maximum rotation frequencies. The former is intuitively obvious to most people: push on a straw and it will buckle sideways. The latter is less obvious, but if you rotate a shaft at its bending natural frequency, it will go unstable and start to vibrate. This is called *shaft whip*, because the shaft can literally whip about. This could damage the shaft, and most likely damage the support bearings and any other components attached to the shaft. Note that both are highly dependent on the mounting condition. Do not assume you have a built-in moment resisting connection unless the shaft is held by at least 3 effective diameters. In addition, use the ROOT diameter of a leadscrew's thread for shaft calculations! What do you do if you are not sure if you have a good moment supporting mount? You could run an experiment and measure the shaft deflection under load to see if it matches the static equations.

Sometimes, as is the case with leadscrews, shafts are placed in tension, and this can increase their natural frequency significantly if the tensile load applied is much greater than the buckling load[1]. In addition, some weight limited machines, such as turbines, operate at super critical speeds, where the machine will start up at a low frequency but then accelerate very quickly to its operating frequency which is above the first natural frequency of the shaft! In these cases, the damping from the support bearings plays a very important role in helping the machine to make the transition. In robot design contests, shaft buckling and rotation instability can be a problem because shafts are often made from slender members. The spreadsheet *shaft_stability.xls* examines various stability criteria for shafts.

Make a list of the shafts that are used, or those that may be needed, in your machine and consider different strategies for mounting and aligning them. What are the buckling and critical speed values for these shafts and are they an issue at all for your design? How might you increase these values if needed?

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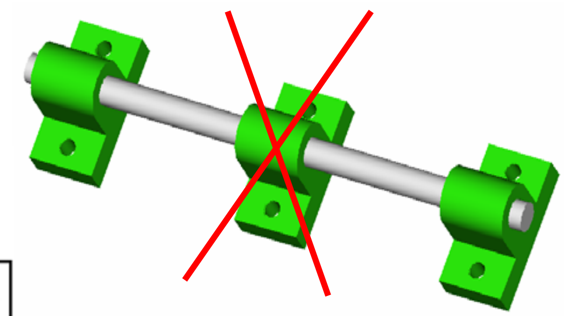
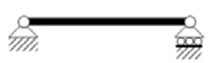
1. See for example Kenneth G. McDonald, *Vibration Testing: Theory and Practice*, 1995, John Wiley and Sons, New York.

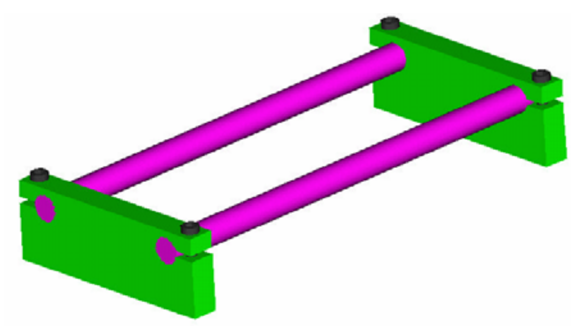
Shafts: *Mounting & Stability*

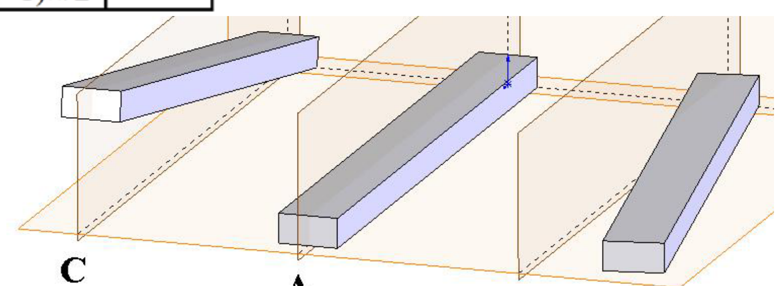
- *Abbe* and *Saint-Venant's* Principles and *Exact Constraint Design* must be carefully considered when designing shaft mountings
- Design the system to accommodate misalignment
  - Allow for clearance between bearing and shaft to accommodate misalignment
  - Use flexible couplings or self-aligning (spherical) bearings
  - Use designs that automatically accommodate misalignment
  - Line-bore holes at the same time through mounting plates



$$\omega_n = k^2\sqrt{\frac{EI}{A\rho L^4}} \qquad F_{\text{Axial buckling}} = \frac{cEI}{L^2} \qquad \Gamma_{\text{Torsional buckling}} = \frac{2\pi E I_P}{L} \quad I_P = \frac{\pi D^4}{32} \quad G = \frac{E}{2(1+\eta)}$$

								
	Cantilevered		Simply Supported		Fixed-Simple		Fixed-Fixed	
mode n	k	c	k	c	k	c	k	c
1	1.875	2.47	3.142	9.87	3.927	20.2	4.730	39.5
2	4.694		6.283		7.069		7.853	
3	7.855		9.425		10.210		10.996	
4	10.996		12.566		13.352		14.137	
n	(2n-1)π/2		nπ		(4n+1)π/4		(2n+1)π/2	





Shafts: *Component Attachment*

The previous discussions focussed on stress concentrations between components, their connections to shafts, and the conditions most likely to result in a robust design. In general, avoid sharp transitions, even between components mounted on the shaft and the shaft itself. When components are attached to shafts, there are often many different types of loads applied, and in order to account for all their effects on the total state of stress in the shaft, (e.g., σ_{axial}, $\tau_{torsional}$, $\sigma_{bending}$) use an equivalent stress criteria such as Von Mises equivalent stress:

$$\sigma_{equivalent} = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2}{2} + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{zx}^2}$$

But what about practical aspects of designing such connections from applications ranging from design contest robots that have to work for a couple weeks to industrial robots that have to work for many years? In general, you get what you pay for, and of course you should only pay for what you need.

Press or shrink fit connections are often most reliable for when they are well designed, the materials behave as if they were monolithic. The biggest issue is manufacturing tolerances, which are often harder to control than the required interference fit. The spreadsheet *Joint_interference_fit.xls* illustrates this quandary, and it is discussed in more detail on the following page. Note that for high torque and high precision production systems, a design engineer may want to select mating tapered polygons[1].

The next option is to use a component that has flexures cut in it and a collar that compresses the flexural element to clamp the shaft. In the case of using a plastic tube as a coupling between two shafts, simple hose clamps achieve this function. A component may also be designed as shown where the end is split and a bolt is used to squeeze the two segments together.

Many components such as gears, however, have a hub but no means to provide a clamped-type connection, other than a shrink-fit. A spline may be used to transfer the torque. The simplest spline-type connection to use, especially for a robot contest, is a hexagonal shaft that fits through a hexagonal hole broached into the component. Other parts of the shaft may be made round on a lathe so they can pass through bearings.

1. See for example http://www.generalpolygon.com/index.htm

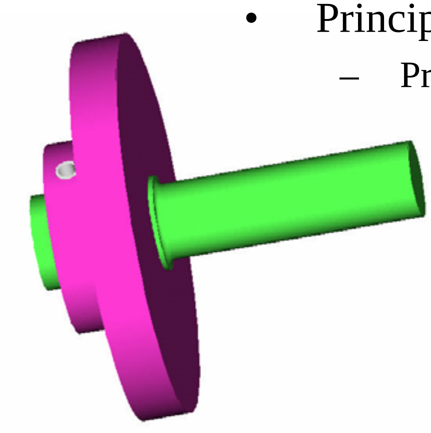
If a spline is not practical, a keyway is often the next best type of connection, and indeed it is one of the most common methods used to transmit torque between shafts and components in industrial applications. Shown are square cross-section keys. The components are often held axially in position using setscrews that push against the keys. However, alternating torque loads can loosen the setscrew by causing fatigue between its tip and the key so thread locking compound should be used. The square keyway, however, has to be milled and it also causes a fairly large stress concentration. A woodruff key is a semicircular key, where the slot cut in the shaft is then cut with a simple plunging rotary cutter[2]. For any key, the torque transmission capability is equal to the product of the key cross sectional area in shear, the radius of the shaft, and the shear strength of the key material.

After a keyway, a pinned connection is probably the most desirable. To keep the pin from falling out, a spring pin is typically used: spring pins are hollow cylinders with one edge slit. The torque transmission capability is then equal to the product of the cross sectional area of the pin in shear (both sides of the pin), the shaft radius, and the shear strength of the pin material. In order to ensure that the hole through the component lines up with the hole in the shaft, the component needs to be placed on the shaft and then the hole drilled through component and shaft at the same time. Remember, the pin needs to be on a free-end of the shaft so the shaft does not see bending loads or it is likely to fail.

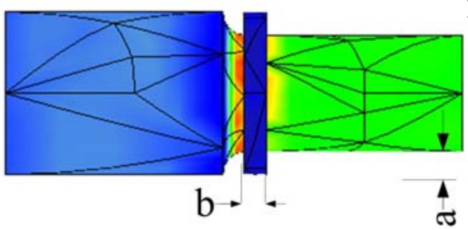
Your worst connection nightmare is just a setscrew pushing on a shaft (either directly or on a flat on the shaft). Such a connection is virtually guaranteed to eventually work itself free, even if you use thread locking compound. So beware, because it is amazing how many products still use this type of connection! For your robot design contest machines, which only have to work for a period of a couple weeks, you might get away with it if you use the point of a drill to form a conical depression in the shaft into which the setscrew seats; thus acting kind of like a small keyway.

2. For a good discussion on types of keyways and how to machine them, see Machinery's Handbook, Industrial Press, New York

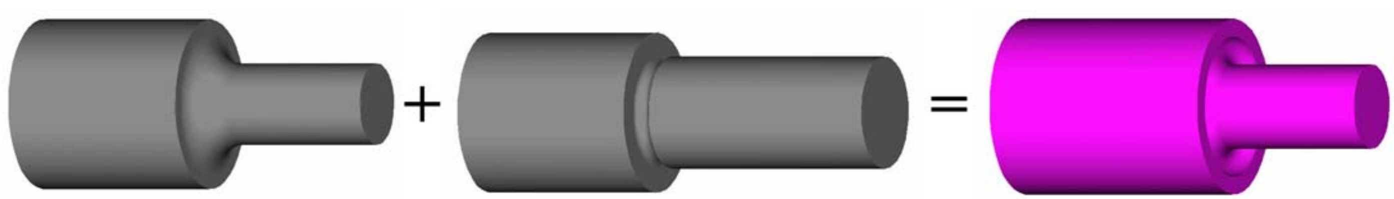
Shafts: *Component Attachment*



- Principal functional requirements for attaching a component to a shaft:
  - Prevent the component from slipping
    - *Interference-fit:* Best torque transmission, but can be difficult to manufacture
      - A variation is when the shaft and hub are ground to have a mating lobed fit
    - *Spline:* Excellent torque transmission, but can be expensive
    - *Circumferential clamp:* Very good torque transmission, low stress concentration, modest price
    - *Key:* Very good torque transmission, modest stress concentration, low cost
    - *Pinned:* Good torque transmission, modest stress concentration, low cost
    - *Setscrew:* Very low cost, but could be your worst nightmare if it loosens!
      - After the first round of competition in 2007 2.007 contest: *"I was wondering if I could use a drill press in lab sometime tomorrow to drill a hole in a gear and shaft for a pin. I realized after tonight that my set screw on my shaft still slips even after making a flat on the shaft"*
  - Minimize stress concentrations
    - Use radiused corners, but be careful of having a square surface for locating components
      - For larger differences in diameters, a radiused undercut can be used
      - For smaller differences in diameters, a small straight flange in front of a radiused corner can be used



	$K_{t\ radius}/K_{t\ mini\ flange}$		
b/a	axial	torsion	bending
0	1.0	1.0	1.0
0.5	1.2	1.4	1.1
1	1.3	1.4	1.1



Shafts: *Interference-Fits*

Properly implemented, an interference-fit connection between components is perhaps the best connection means. This is due to the fact that there is no mechanism to loosen with time; however, because there is no mechanism, there is no means for adjustment, and hence care in manufacturing is critical. This is especially true given that the amount of interference required to achieve a desired interface pressure is very small, so the amount of interference required to yield the material is also small.

When designing a connection between objects, one must be careful to consider all the forms of loading that may occur, and then find or develop simple analytical models for each. Superposition allows them to be combined such that an estimate can be obtained for the total state of stress in the joint. For a interference-fit connection between a gear and a shaft, loads that must be accounted for include: torsional, axial (some forms of gears generate axial loads), centrifugal expansion, and differential thermal expansion. In addition, bending loads may also be present, but it is the former that most affect the joint structure.

In order to create a spreadsheet or a program to determine the stresses, the independent and dependant variables must be identified. In an ideal case, all the loads are entered and the program directly or iteratively solves for the desired parameter, such as the required (interference-fit) interference. The analysis can also proceed from a manufacturing perspective where reasonable dimensions are entered and the results are reviewed. In the case of a interference-fit, the latter is chosen here and is indeed a more practical method. To start, the spreadsheet on the opposite pages has sections for entering in the known and assumed parameters for the design. The inner and outer diameters of the bodies are entered along with manufacturing tolerances.

The first calculation determines the minimum interface pressure required to carry the loads so the interface does not slip. Next, the pressure caused by the interferences must be determined. Finally, the stresses and radial displacements caused by the various loading conditions must be determined. All of these different calculations are performed on the spreadsheet, where variable names are assigned to the spreadsheet cells, so the equations entered appear as decipherable mathematical expressions and not as incomprehensible strings of cell references! With reasonable descriptors, a well-written spreadsheet should need little or no additional documentation.

Torsional and axial loads are very straightforward to model, where for a diameter D, the stress caused by the torque G and force F are easily determined. In addition, however, the poisson contraction (or expansion) in the radial direction from the axial load is also determined:

$$\tau = \frac{32r\Gamma}{\pi D^4} \qquad \sigma_z = \frac{4F}{\pi D^4} \qquad u_r = \frac{r\eta\sigma_z}{E}$$

Given an interference Δ, the outer body modulus of elasticity E_O and outer diameter D_O, the inner body modulus of elasticity E_I and inner diameter D_I, and the nominal interface diameter D_{int}, the resulting interface pressure is[1]:

$$P = \frac{\Delta}{D_{int}\left\{\frac{1}{E_O}\left(\frac{D_{int}^2 + D_O^2}{D_O^2 - D_{int}^2} + \eta_O\right) + \frac{1}{E_I}\left(\frac{D_I^2 + D_{int}^2}{D_{int}^2 - D_I^2} - \eta_I\right)\right\}}$$

Once the amount of interference is determined, it can be determined if the component can be heated to expand and then placed over the shaft. If a reasonable temperature can be used, then a *shrink-fit* is most desirable. Otherwise a *press-fit* can be used where a shaft is pressed into the hole, but with care to prevent the shaft from buckling.

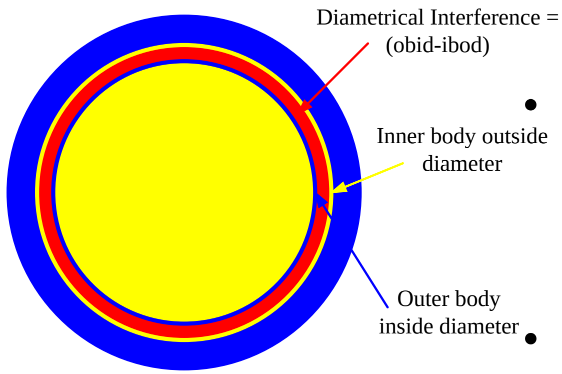
It only takes a little bit of interference to create a strong joint, but it only takes a little bit of tolerance error to make it not work. Depending on the materials, they have to have significant ductility else they will crack when forced together. The interference-fit can sometimes be made much larger to ensure yielding of the materials, but make sure that the interface pressure at yield is enough to transfer the desired loads. Look at the critical outputs highlighted in yellow on the spreadsheet *Joint_interference_fit.xls* as they help to indicate if the design is good.

1. From Slocum A.H., Precision Machine Design, Prentice Hall, Englewood Cliffs, New Jersey 1992, pp 387-399, or for For a more detailed discussion, see S. Timoshenko, Strength of Materials, Part II, Theory and Problems, Robert Krieger Publishing Co., Melbourne, FL.

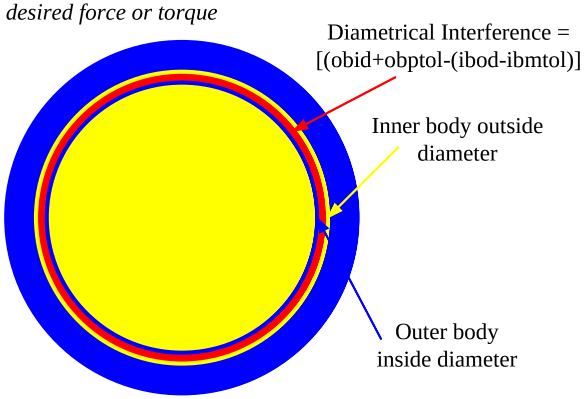
Shafts: *Interference-Fits*

- Interference-fits: most reliable connection:
  - *Joint_interference_fit.xls* models tolerances, differential thermal expansion, and high speed rotation
- Manufacturing issues are of greatest concern
  - Interference is typically small to prevent too high stresses:
    - Radial or circumferential stresses
    - Buckling while pressing in the component
  - Tolerances may cause too high stresses, or too low an interface pressure
  - Clean, high friction interface

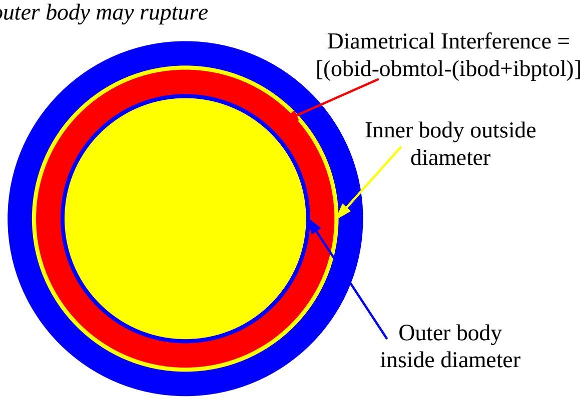
Ideal case



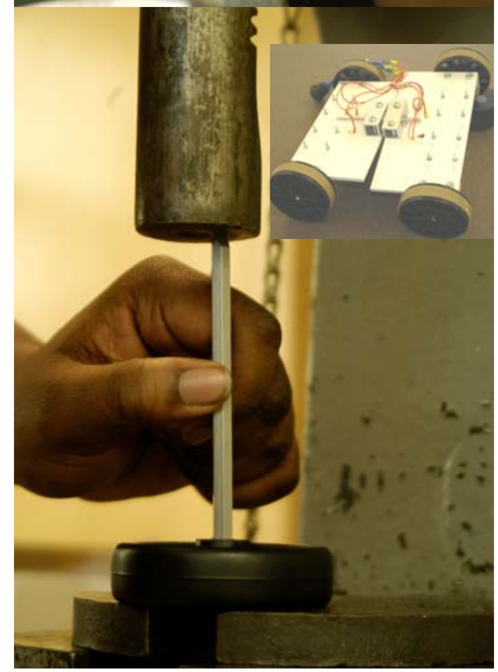
Worst case for loose fit:
Joint may not be able to transmit desired force or torque



Worst case for tight fit:
Yield stresses may be exceeded and outer body may rupture



Required interference parameters	
Minimum required interface pressure, rPI (N/mm^2)	**0.8017**
Add Poisson diametral interference, ddp (mm)	**6.92E-06**
Add differential thermal expansion diameter, ted (mm)	**7.50E-04**
Add outer body rotating radial displacement, robd (mm)	**1.80E-06**
Subtract inner body rotating radial displacement, ribd (mm)	**4.17E-07**
Total additional interference amount to be added to ibod, addi (mm)	**0.0008**
Interference fit calculations (assumes addi has been added to ibod)	
maximum interference, maxdelta (mm)	**0.0078**
Maximum resulting interface pressure, maxIP (N/mm^2)	**26.36**
minimum interference, mindelta (mm)	**0.0038**
Minimum resulting interface pressure, minIP (N/mm^2)	**12.77**
Minimum safety margin (min obtained pressure/required pressure)	**15.9**



Hexagonal shaft press fit into a plastic wheel's round bore for high torque transmission

Couplings

Couplings are required between rotary and linear actuators and driven components because the actuators are intended to move in one degree of freedom (linear or rotary) but they can never be perfectly aligned. As a component moves, it will not always be aligned with the actuator. The product of the net difference in motion with the stiffness of the connection between the two systems yields the misalignment force on the actuator. If a rigid coupling is used, the forces can be extremely high, and something, usually the bearings, will have to give and they will soon fail. Furthermore, a significant portion of the system's power can be expended in the process. When a coupling is ideal, and only restrains the intended motion with negligible effect (stiffness) in all other axes, it is said to be a *non-influencing coupling*.

There are three strategies for coupling designs. The first is to leave clearance between geometric features so the motion occurs across a sliding bearing interface without causing large bearing loads. A linear example is a clevis at the end of a hydraulic cylinder, where there is clearance between the clevis pin and the bore into which it fits. A rotary example is an *Oldham* coupling which is two comprised of two cylinders, each with a slot cut in their faces, each attached to the ends of two shafts that are facing each other. A plastic disk with raised rectangular features on each side is placed between the cylinders. Small motions between the shafts are accommodated by the plastic keys in the slots. When more torque and misalignment capacity are needed with a more linear response, four jaws on each cylinder face each other and then an eight-armed plastic *spider* insert acts as the interface between the two.

The second strategy is to use a connection that is only stiff in the desired direction. An example is a tube with circumferential slits cut in it (a *helical* coupling). Merely using a piece of plastic tubing is not acceptable because if it is short enough to provide enough torsional rigidity, it is often not sufficiently radially compliant. Super precision instruments may go to extreme measures and use *taut wire couplings*.

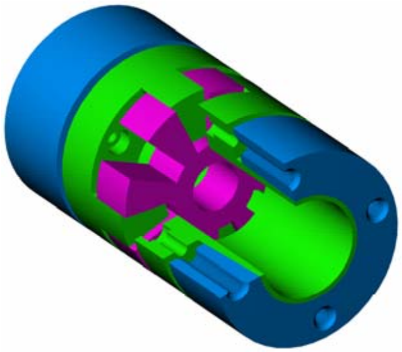
The third strategy is to use linkage elements (linear and or revolute joints) to create a mechanism that transmits the appropriate force or torque. A common example of this type of strategy is to use universal (*Hooke's*) joints at the ends of a drive shaft. Precision instruments often use *wobble pins*, which are pins with conical ends that rest in a spherical cup to transmit forces.

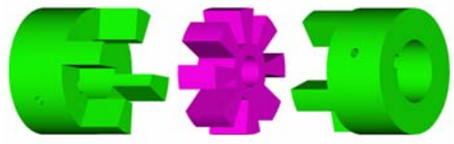
It is important to identify the degrees of freedom that are to be constrained, and the accuracy that is to be maintained. Draw the actuator output and the element to be moved. Attach the two with a coupling element that is only stiff in the direction in which force is to be transmitted. For example, when a leadscrew nut is attached to a carriage, since the leadscrew is not perfectly straight or radially stiff, it may act as its own coupling, allowing the nut to be attached directly to the carriage. However, as the leadscrew turns, it will impart an oscillating lateral force onto the carriage. If the nut were attached at the end of the carriage, it would cause the carriage to pitch, which would cause Abbe effect errors! On the other hand, if the nut is attached at near the carriage's center of stiffness, then radial error motions of the screw will not cause pitch errors.

With respect to rotary motion systems, it is important to note that simple pin-in-slot type couplings, such as Oldham and Spider couplings, and simple universal joints induce sinusoidal error between the actuator and driven shaft. The maximum error ε_θ between the angular position of the input shaft and the output shaft is just the eccentricity error between the shafts divided by the coupling radius. For shafts that are angularly misaligned, the error ε_θ between the angular position of the input shaft and the output shaft is about equal to the product of the misalignment angle and the sine of the rotation angle. At a maximum, the error is on the order of the shaft misalignment error. Couplings that use elastic compliance between elements have negligible following error. Constant velocity universal joints are also available.

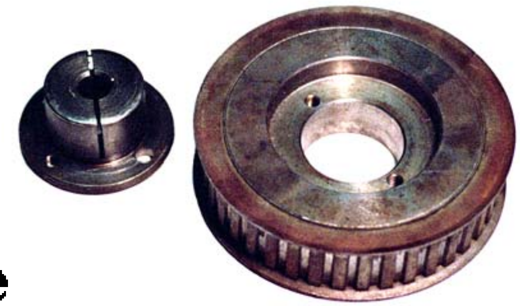
Finally, consider how the couplings are attached to the shafts. A circumferential clamping attachment is the best alternative (e.g., an Oldham coupling). Setscrews are to be avoided, but if they must be used, also use a keyway. If a keyway cannot be used, use a thread locking adhesive and prayer.

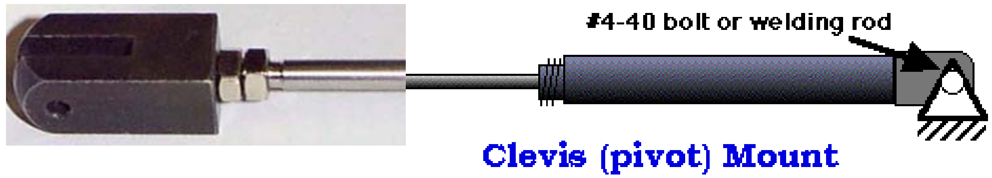
Complete an inventory of all the interfaces between moving elements in your machine and assess the coupling strategies you could use. What price do you pay (materials, time) for what performance do you expect to get? Are you sure your couplings are strong enough and will provide the desired accuracy or repeatability? Are you sure that they provide the degrees of freedom needed to ensure that your actuators are not wasting valuable power deforming elements that should not need to be, nor want to be, deformed!



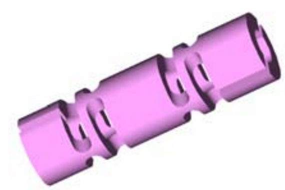


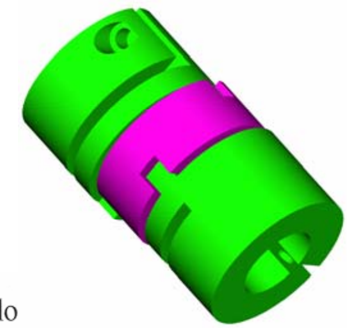
Couplings

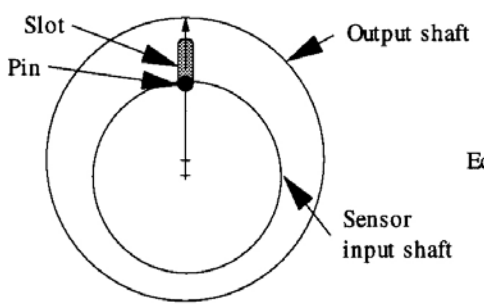


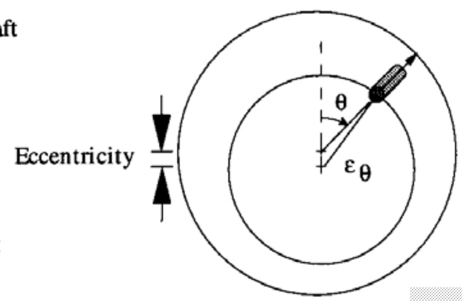


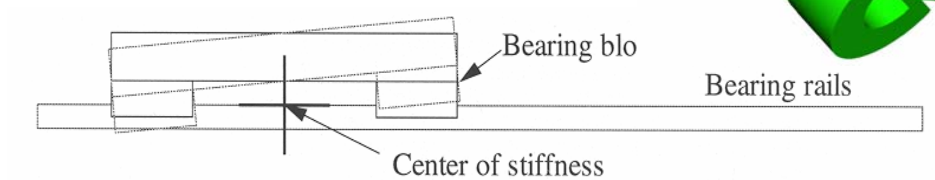
- No two moving components can be perfectly aligned
  - To prevent over constraint from destroying components and robbing your system of power, use couplings
- Identify the degrees of freedom desired, the accuracy (repeatability) needed, and the load capacity and stiffness required in the direction of force or torque transmission
  - Select either a sliding, flexural, or rolling element coupling
  - See Page 10-21 for life calculations

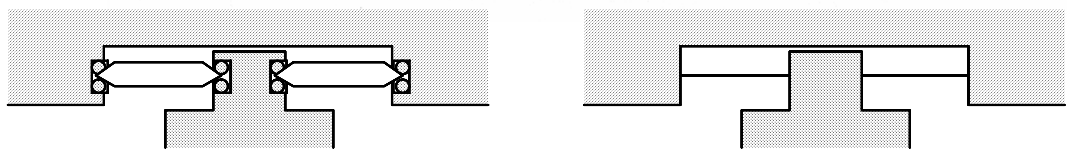


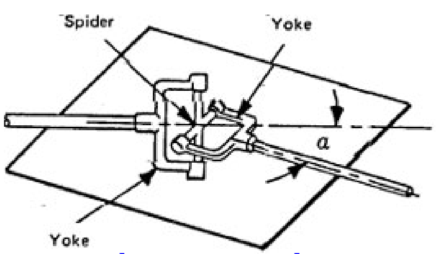


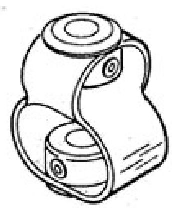


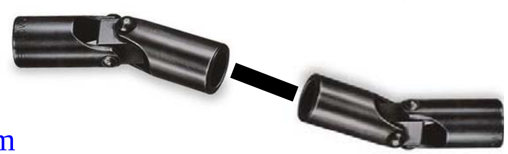












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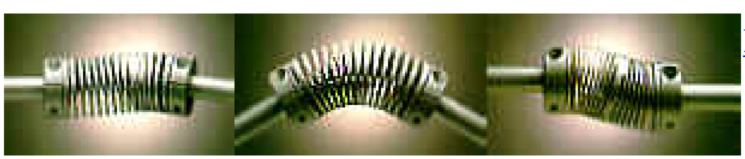
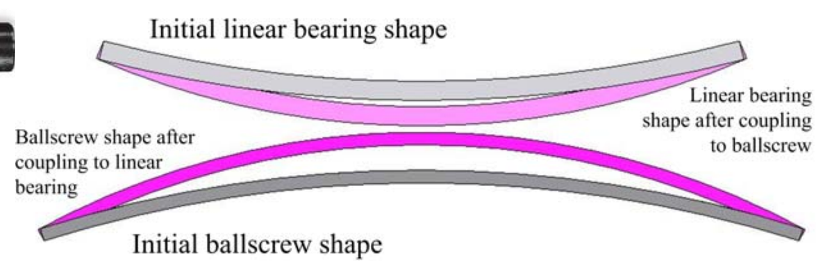


http://www.heli-cal.com/



Couplings: Cheap & Easy Example

Most of the couplings described above need to be purchased, but for a robot design and build contest, this is often not possible; thus how can you morph all the above into a low-cost effective coupling? Shafts in a simple robot built for a contest are likely to be misaligned in 5 degrees of freedom: hence, the coupling must allow for radial (X, Y), axial (Z), and pitch (ε_x) and yaw (ε_y) misalignments. Before starting to sketch ideas, consider the boundary conditions of the problem in terms of available materials and manufacturing processes:

- Materials: pins (straight or spring pins), and round stock
- Manufacturing: drilling, milling, sawing, turning

Recall that there are two basic types of joints, linear and rotary, and that the motions to occur are of one shaft *relative* to the other shaft. This implies that some of the motions can be on one shaft, and some of the motions can be on the other shaft. It is also worth noting in the thought process, that the X and Y motions required are orthogonal to each other. Furthermore, the pitch and yaw motions also occur about orthogonal axes. This if we develop a joint that can allow linear motion and rotation about that axis, then we could just use the same joint, rotated ninety degrees on the other shaft and we would have taken care of four of the five degrees of freedom. This leaves the Z-direction, which intuitively we can provide for with an axial slot in the coupling body.

Assuming we use a simple axial slot, how can we provide for motion along the slot (for the Z motion) but also in and out of the slot? The answer would seem to be a simple pin that is pressed into the shaft (or glued, just something to keep it from falling out). If the slot is cut into a tube whose inner diameter is larger than the shaft, then the radial clearance between the two is the radial motion that will be accommodated by the coupling. Just make sure to make the length of the pin equal to the outer diameter of the coupling plus the diametrical clearance between the shaft and the coupling's inside diameter.

As the sequence of images shows, voila! The coupling is born. However, what are the risks and appropriate countermeasures? Will the coupling body flop down and rattle? How fast will it be turning? Can soft sponge rubber be used as a spacer between the shaft and the coupling body to keep it nominally centered? What about a wide small diameter rubber band or similar elastic sleeve? What about bending of the pins?

Bending of the pins? How can we model that? The forces on the pins can be modelled by assuming the torque transmitted acts as a couple across the diameter of the coupling body; thus the bending force on a pin is equal to the torque divided by the diameter of the coupling body. The spreadsheet Cheap_coupling_design.xls performs the calculations. Why is the torque that can be transmitted independent of the coupling body diameter?

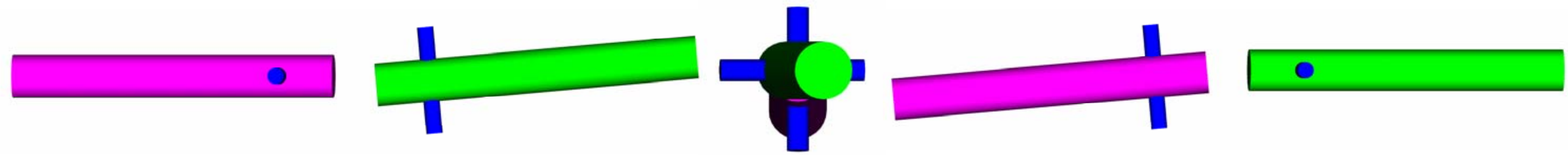
Inputs in **BOLD**, outputs in **RED**	
Shaft diameter, ds (mm)	**4**
Coupling body outer diameter, OD (mm)	**8**
Coupling body outer diameter, ID (mm)	**6**
Pin diameter, dp (mm)	**1.5**
Cantilever length, L (mm)	**2**
Max. allowable stress, maxs (N/mm^2)	**200**
I/c, Ioc (m^3)	**0.33**
Bending moment, Mb (N-mm)	**66**
Resulting torque transfer (Mb from both sides) (N-mm)	**133**

What about the forces between the pin and the slot causing the pin to dent the slot? This would be a Hertz contact pressure failure mode, and is discussed in greater detail in Chapter 9 (read forward!), but suffice to say that the theory and a spreadsheet to predict the contact pressure are provided!

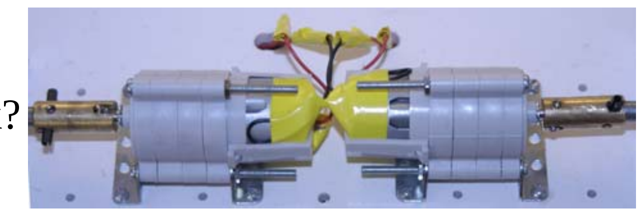
A molded plastic cheap and easy coupling should perform more than adequately for most situations. If you determine that for high impact or high torque applications you need a metal version, how would you manufacture it?

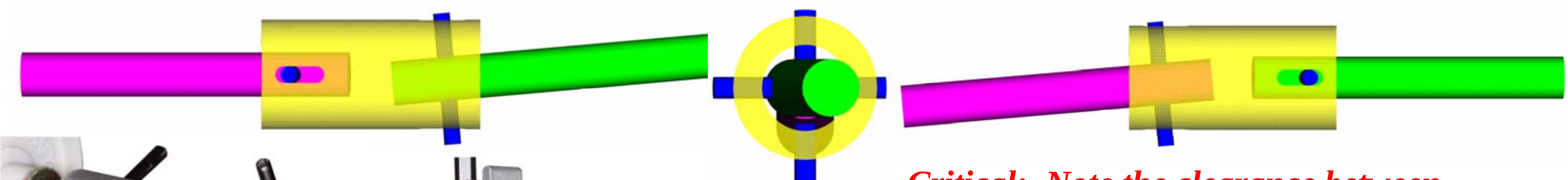
Couplings: *Cheap & Easy Example*

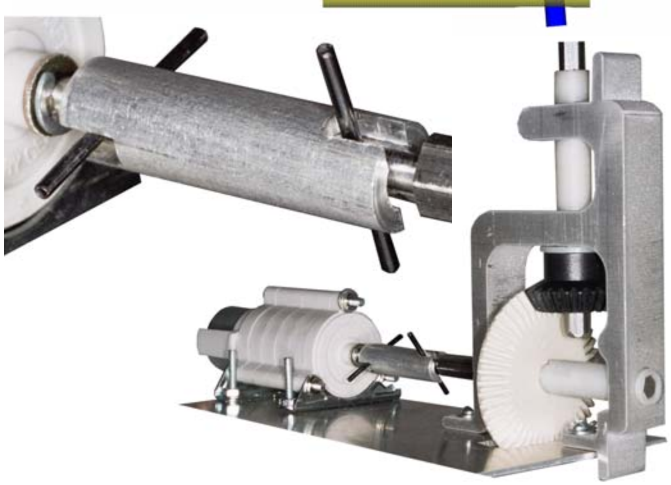
- What about in a robot design contest where two shafts may be linearly misaligned axially, vertically and horizontally, and angularly misaligned?



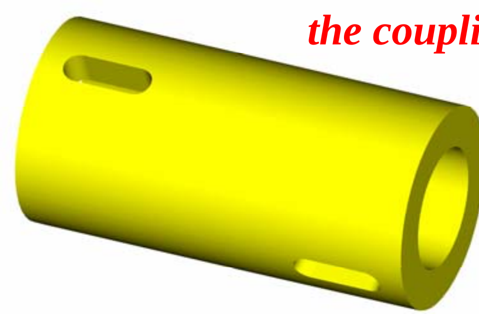
  – How can you design a simple one-piece coupling to enable one shaft to transmit torque to the other shaft?
    - How strong must be the radial pins (*coupling_low_cost.xls*)
    - Can the coupling be made from plastic tube to reduce shock?
    - Would O-rings be useful to nominally center it?

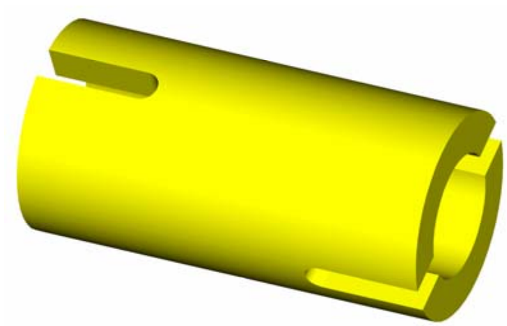






Critical: Note the clearance between the coupling bore and both shafts!





Topic 5 Study Questions

Which suggested answers are correct (there may be more than one, or none)? Can you suggest additional and/or better answers?

1. Pulleys can be used to direct or amplify the force in a cable:
   True
   False
2. The amplification of force in a cable wrapped around pulleys is equal to the number of strands of cable between the pulleys and the number is always even:
   True
   False
3. A capstan is typically a fixed, or controlled rotation, body-of-revolution which a cable wraps around
   True
   False
4. A capstan can be used in a band brake, where a band is anchored to a structure, and then wraps around a shaft:
   True
   False
5. A cable wrapped around a capstan by θ radians with coefficient of friction μ and being held with a force F_{hold}, can resist the pull of a cable with many times higher force $F_{pull} = F_{hold}\, e^{\mu\theta}$
   True
   False
6. If a pulley's outer diameter is 3-5 times larger than the shaft on which it is supported, then a sliding contact bearing can provide 90+% efficiency:
   True
   False
7. If a belt or cable runs around a fixed shaft, then there is a lot of friction between the belt and the shaft, and the efficiency is low:
   True
   False
8. Belts & Cables can be used to convert rotary to linear motion, where the force F in a belt with tension T on a pulley of diameter D that can be generated by the torque Γ can be conservatively estimated by:
   $F = 2\Gamma/D$ for toothed belts
   $F = 2\Gamma/D$ for flat belts but is limited by the capstan effect
   $F = \Gamma D/2$ for all belts
9. Tracks only help when there is loose media or a surface into which they can dig and they do not help on smooth surfaces unless they are covered with a dust layer
   True
   False
10. Tracked vehicles skid steer, and as a result there are large normal forces on the tracks which try to pull them off the drive and idler sprockets
    True
    False
11. A central ridge formed by an idler roller in the middle of the treads can help a tracked vehicle steer
    True
    False
12. Two pulleys' axes of rotation can never be perfectly parallel, so a flat belt will want to drift off (tracking) so pulleys must be crowned (rounded profile) to keep a belt from walking off
    True
    False
13. Pulley tension must be maintained either by proper pulley center distance and belt elasticity, or a mechanism to tension the pulleys or a pulley that pushes sideways on the belt
    True
    False
14. Chains are a very robust means to transmit very large forces and torques without worrying about slip between the chain and the sprocket:
    True
    False
15. Rotary-to-rotary motion is very common (look at your bicycle!) and Rotary-to-linear motion can be obtained by laying a chain on a surface,