FUN*da*MENTALS of Design

Topic 7

Power Systems

Power Systems

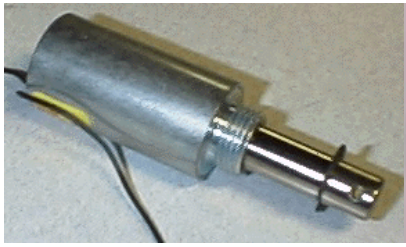
In this book, the first four chapters focussed on developing strategies and overall concepts for machines with motions to achieve them. Before delving into more mechanical detail, it is important to consider how the intended motions will be powered. In fact, when creating a machine concept, one of the most fundamental, and limiting, issues is power: How can power best be stored, used, and controlled? Perhaps most importantly, once you understand the types of power systems that are available, how can you best manage power as a precious resource for your machine?

The first step is to consider the different types of motors that are available. A motor is a device that converts stored energy into mechanical work. The energy may be electrical, mechanical, or chemical. The output of a motor often goes into a device such as a linkage, gearbox, or leadscrew[1]. There are many types of motors, far too many to discuss in detail here, including the very motors of life itself.[2] Hence this chapter will consider simple electromagnet lifting systems, dc brushed motors, solenoids, and pneumatic cylinders. It is assumed that the reader will then be psyched to learn about other types of actuators for more advanced designs!

The next step is to consider energy storage methods, which include mechanical springs, electric batteries, and compressed air bottles. Umbilicals, which deliver electricity and compressed air, may also be available. The control of energy into an actuator will also directly affect machine performance. For example, a motor can be controlled by a mechanical switch that either turns it on or off (bang-bang control), or a proportional controller that enables you to control the speed of the motor.

With an understanding of these basic issues, the design engineer can start to imagine how the machine might be powered. In order to allocate scarce resources, the design engineer must create and manage a *power budget* for the machine. The power budget keeps track of all actuators' power needs as a function of time to make sure that demand does not exceed supply!

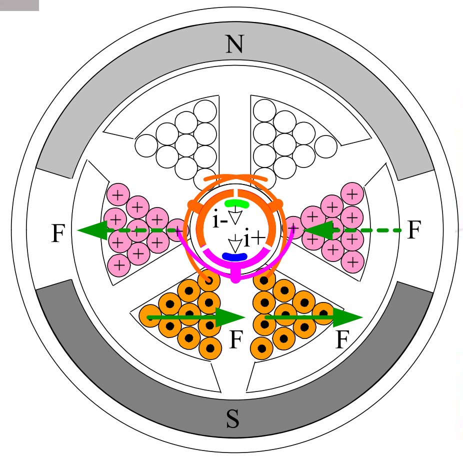
1. These power transmission devices are discussed in Chapters 4-6.
2. See for example, T. Atsumi, "An Ultrasonic Motor Model for Bacterial Flagellar Motors", J. Theo. Biol. (2001) 213, 31-51

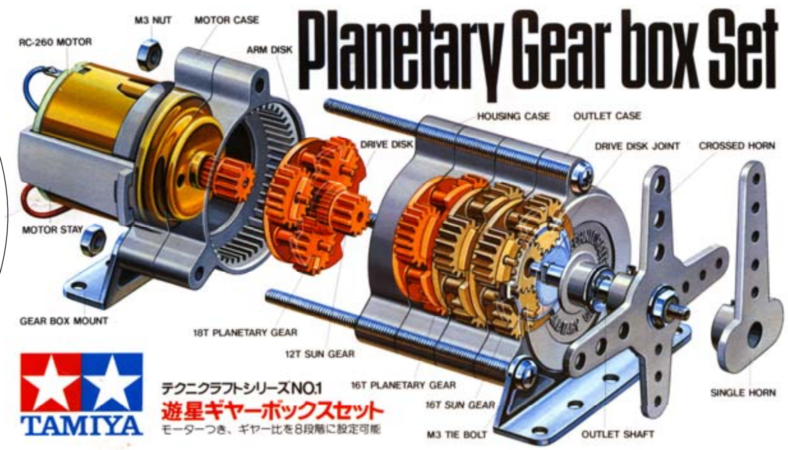


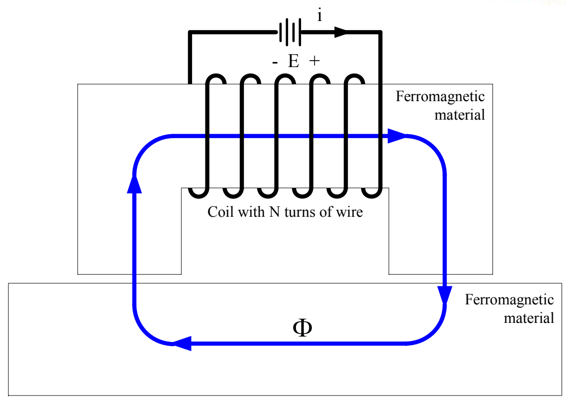
Topic 7
Power Systems

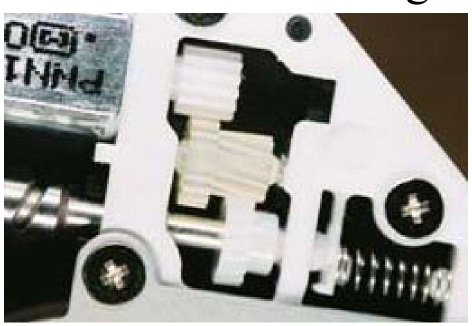
Topics:

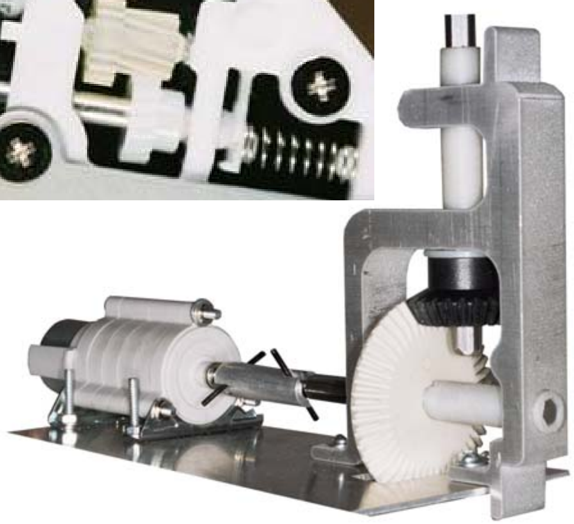
- Systems Engineering
- Electricity & Magnetism
- Magnetic Circuits
- Electric Motors
- Energy Supplies
- Pneumatic Systems
- Power Budgets

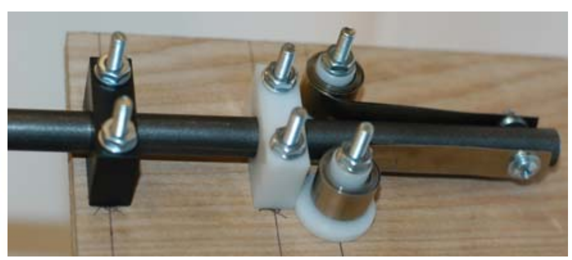












Systems Engineering

The introduction to this chapter introduced the idea of systems engineering. Imagine that a group of engineers has gathered for dinner to discuss the design of a system on which they are all working, and there is only a small table available at the cafe. If every engineer orders soup and salad as an appetizer, will there be space for all the plates? As one engineer reaches for the bread, will he knock over another's glass? What happens when the entrées arrive and the soups and salads are not yet finished? Who will give up their dish?

A system is created from various resources. Resources, from space to power for machines and from time to money for the teams that design the machines, are finite and must be carefully managed before a project starts and while it is running. For example, *power budgets*, discussed at the end of this chapter, are crucial to managing the total energy resources of a system. Indeed, a key to managing resources and successfully creating a machine (or any product) is to minimize duplication of effort, and indeed to share responsibilities. This is true for the design team as well as for the machine itself. In fact, *Occam's razor* can be used to trim the fat from all types of systems. Simplicity is the best place to start a design.

A key to system simplicity is the use of modules. A module may be designed with a rigid frame to enable it to be tested by itself on the bench. The frame of the machine, meanwhile, is also designed to be rigid so the machine can be built and tested as each module is attached. Does this mean that all the modules together will be too rigid and heavy? A system's approach could be to design the module's frame to be just strong enough to carry it from the bench to the machine, where it can then be attached and reinforced by the machine frame. On the bench, a model section of the machine frame would be employed to enable the module to be tested by itself.

As another example, consider the *Axtrusion* design shown on page 1-23. The mechanical complexity of the machine was greatly reduced by using the attractive force of the linear electric motor's permanent magnets to also preload the bearings. This is a great idea, but also take note of the fact that the interdependence of elements on each other can have second order effects that if not carefully considered, can lead to unwanted surprises. In the case of the Axtrusion, it was suspected, and indeed verified, that as the motor coils pass over the magnets, there is a small variation in the attractive force. This varying force affects the preload force applied to the bearings, which causes "error motion" displacements. Even so, the error motions are small enough for many applications.

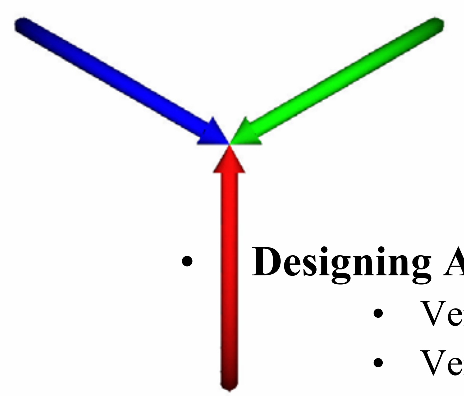
Consider a robot for a design contest, where the first several seconds often decide the course of the rest of the contest. Triggers are often used to control the sudden release of energy from springs that can launch projectiles or the machine itself into action. Triggers can be actuated by solenoids; however, this takes electrical power and a signal channel. What might be the systems approach? The strategy is to look for some other means to provide a momentary release of power. A concept would be to tie the trigger to some other axis, such as a drive wheel, so when the machine starts to move, the trigger is actuated.

In addition to sharing function, the packaging of individual elements and modules can have a profound effect on overall system performance. One of the most significant fundamental issues in this regard is the placement of actuators with respect to the centers of mass, stiffness, and friction. For example, if a machine element (e.g., a leadscrew nut) is located at the center of stiffness, then error motions of one machine element (wobble of the screw) will not cause pitch errors (Abbe errors) in another element (carriage)

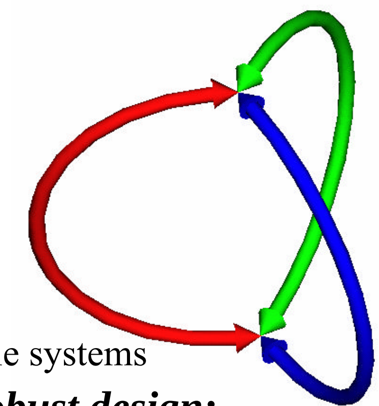
The term *robustness* generally has a primary and a secondary meanings for systems: Can the system tolerate variations and still perform as desired, and can changes (substitutions, repairs) easily be made to a component or module. Good packaging is often an indicator of a robust system.

If You Give a Mouse (engineer) *a Cookie* (task), think of all the other assorted tasks with which you will have to manage. So remember *The Little Red Hen*, and be careful to first prepare the field and then to sow the seeds of success during the planting season, which is long before the harvest!

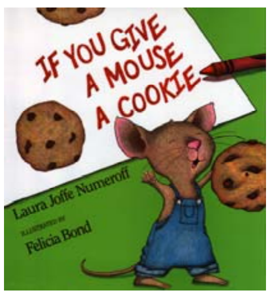
List all the potential elements of your system, and estimate the forces and distances over which they must act, and at what times they are applied. Create a chart of power required as a function of contest time, and integrate to determine the total amount of energy required. Do you think the machine's power system (e.g., batteries) is up to the task? This is your preliminary system power budget.

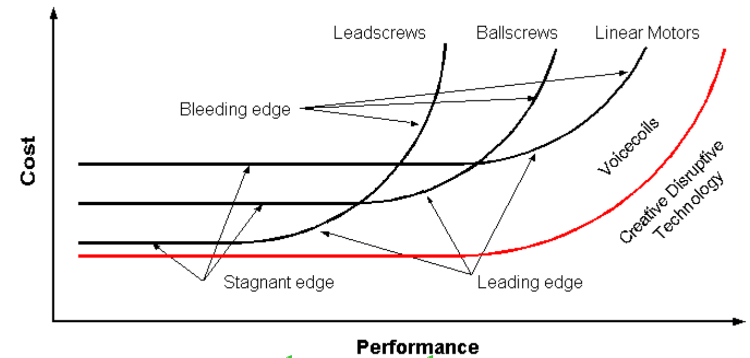


Systems Engineering

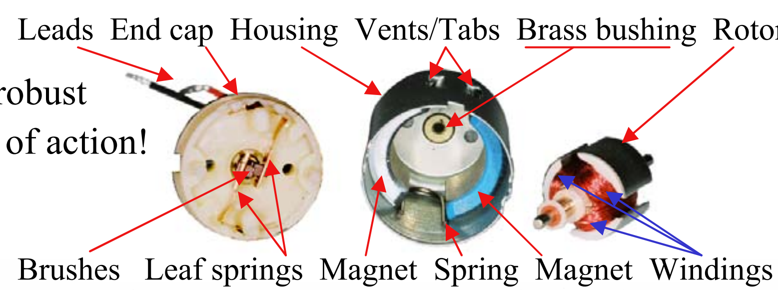


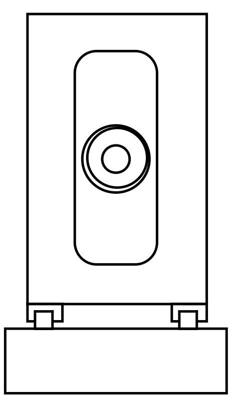
- **Designing Actuators into/for a machine requires a system's approach**
  - Very few things are as simple as they may seem: *The details matter!*
  - Very few things are so complex that they cannot be decomposed into simple systems
- **Three fundamental elements, *blended in appropriate amounts*, yield a *robust design:***
  - **Mechanical**
    - The interface to the physical world
  - **Electronics/Controls**
    - Sensors gather data and send it to the software
    - Amplifiers receive signals from the software
    - Motors receive power from the amplifiers
  - **Software/Strategy**
    - The logic of how the device processes inputs and creates outputs to the control system
  - *The design process for each element are very similar!*

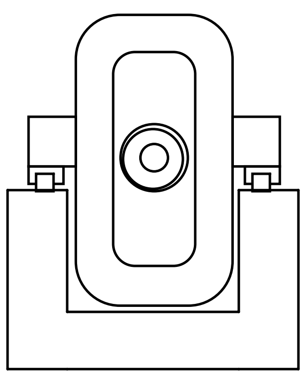


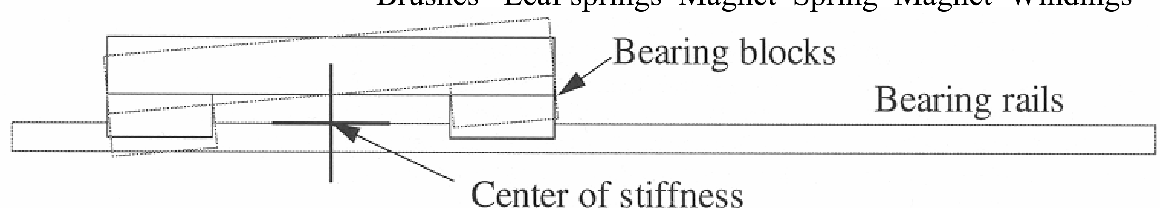


- **Design for Robustness!**
  - A deterministically designed system is likely to be robust
    - Actuators should be mounted near the centers of action!









Systems Engineering: *Transmissions*

The transmission transmits power from a motor to an actuator or mechanism that does useful work. Transmissions are critically important because they transform high speed low torque power from a motor (voltage is cheap, current is expensive) into the low speed high torque power needed by most mechanisms. Motors are generally less efficient at low speeds, so it is the system combination of motor, transmission, and mechanism that enables a machine to operate efficiently. Transmission design includes: determining the best motion profile, calculating the power the system will have to transmit, determining the best or "optimal" transmission ratio, and checking that the velocities and efficiency are within reason. In general, sliding contact systems (screws, sliding bearings) have efficiencies only on the order of η = 30%, and rolling contact systems (gears, ball bearings, hard wheels) have efficiencies on the order of η = 90% and sometimes higher. Overall efficiency for many well-designed and built robot contest machines is typically on the order of 50%.

The fastest way to move is to have constant acceleration (and deceleration) which results in a triangular velocity profile; but this is like stepping on the gas and then stepping on the brakes so it is only used when travel time is to be minimized. Often, a motor accelerates a load up to a maximum speed at which the motor can operate, and then the load moves at a constant velocity until it is then slowed by the motor. Consider a vehicle of mass m which is to move a distance D with a maximum acceleration a, lest the wheels spin. If the time for constant acceleration a (and deceleration) is t_a, and the total time of travel is t_c, then the ideal (no losses) relationships are:

$$D = \frac{v_{\max} t_a}{2} + v_{\max}\left(t_c - 2t_a\right) + \frac{v_{\max} t_a}{2} = a t_a \left(t_c - t_a\right) \qquad v_{\max} = a t_a$$

$$v_{\max} = \frac{D}{\left(t_c - t_a\right)} \qquad a = \frac{D}{t_a\left(t_c - t_a\right)} \qquad P_{\max\_power} = \frac{mD^2}{t_a\left(t_c - t_a\right)^2}$$

$$W_{total\_work} = \int F v dt = 2m\int_0^{t_a} a\left(at\right)dt = \frac{mD^2}{\left(t_c - t_a\right)^2}$$

This analysis can help plan the time to move a given distance, for example, by either a triangular or trapezoidal motion profile. The graph shows that the velocity profile with the lowest peak power is trapezoidal and the acceleration, constant velocity, and deceleration times are equal[1] so γ = 3. It has been shown by Tal[2] that if a parabolic profile is assumed to represent 100% efficiency in a particular application, a trapezoidal profile with equal acceleration, slew (constant speed), and deceleration times is 89% efficient, and a triangular profile is 75% efficient.

The equations show that if the time to move a desired distance is cut in half, both t_a and t_c are reduced by a factor of 2, then ***EIGHT*** times the power is required! Thus one must be very careful before one casually plans to cut move times, which could require a total redesign of the machine. Furthermore, this analysis also assumes that the system is dominated by the maximum acceleration, which is limited, for example, by wheel slip. Maximum DC motor power is generated at one-half the maximum speed, hence it would have to be conservatively assumed that the motor was sized such that one half its peak torque is used to generate the maximum acceleration.

Here the design of the power transmission system depends on how much design latitude exists in the system. For a robot contest, one should not stall the motor or spin the wheels, so the above analysis is conservative. For a product to be sold or used in a competition that stresses energy efficiency, the "exact" motion profile equations would be used to maximize system efficiency, and more parabolic profiles would likely be used. For a machine where a period of constant velocity is needed to perform some function, such as cutting, then power will be sacrificed to get the system up to speed as quickly as possible in order to enable the machine to spend more useful time performing the process. Once the move time and velocity are known, together with the motor speed, the "optimal" transmission ratio can be determined.

What is the mass of your robot? How far must it travel and in what time? What is the coefficient of friction between the wheels and the surface? Can you glue sandpaper to the wheels? How does move time affect your scoring potential? Do your motors have enough power to move your robot the distance it needs to go in the time you have? Experiment with *Power_to_Move.xls*

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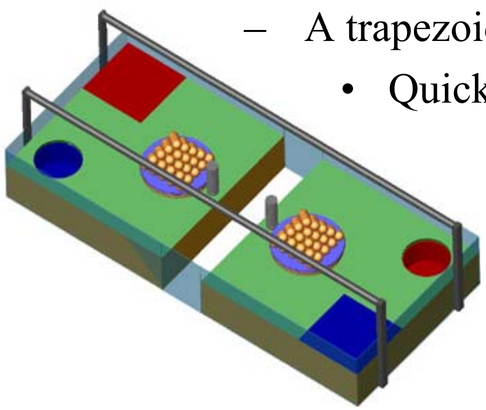
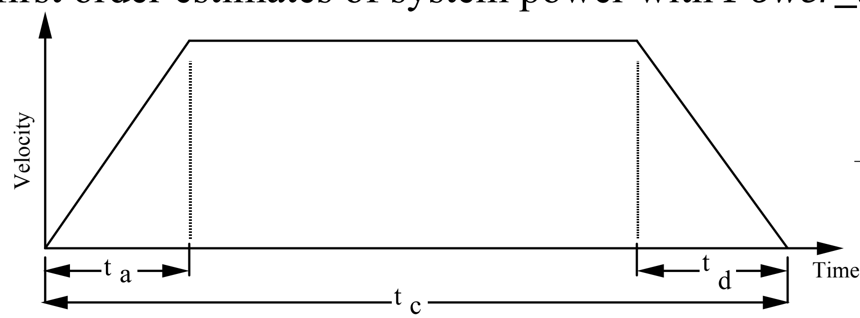
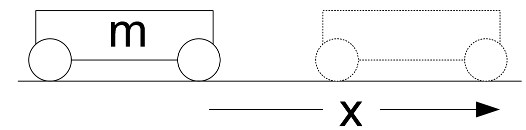
1. It is assumed that the motor issued for slowing the load, in the form of a reverse torque, and hence it consumes power.
2. J. Tal, "The Optimal Design of Incremental Motion Control Systems," Proc. 14th Symp. Increment. Motion Control Syst. and Dev., May 1985, p. 4.

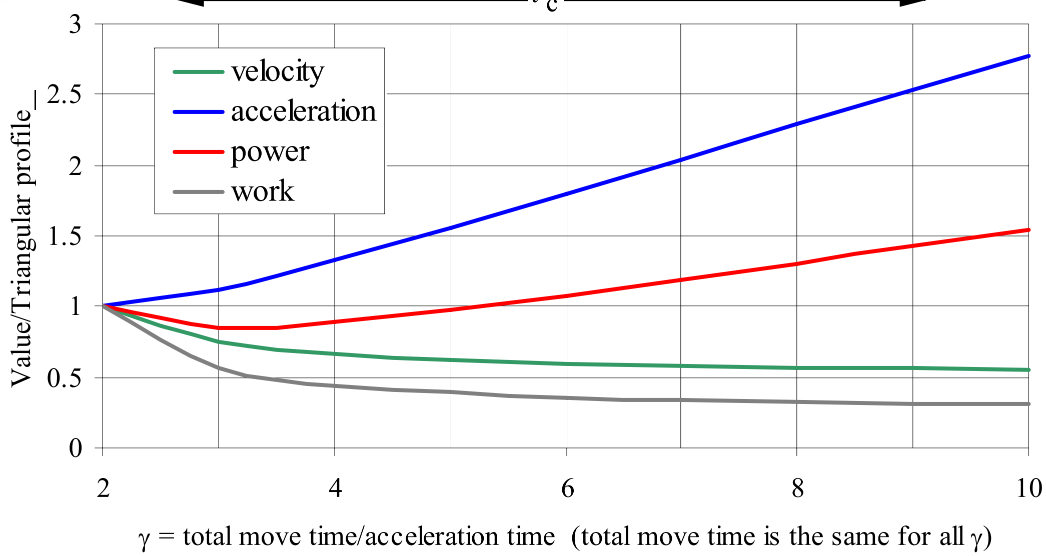
Systems Engineering: *Transmissions*

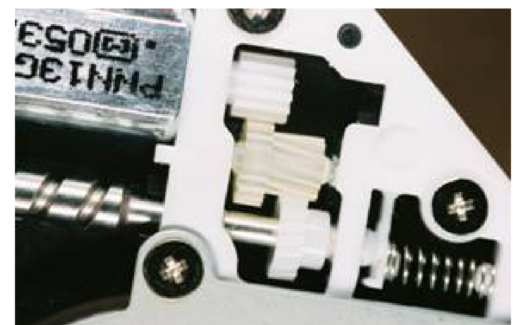
- A transmission is used to convert power from one form into another
  - E.g., rotary to linear motion, low-torque high-speed to high-torque low-speed
- To move a specified inertia m (J) a specified distance D in a given time t_c, with an acceleration time t_a
  - A trapezoidal velocity profile is generally very efficient
    - Quick first order estimates of system power with *Power_to_Move.xls*

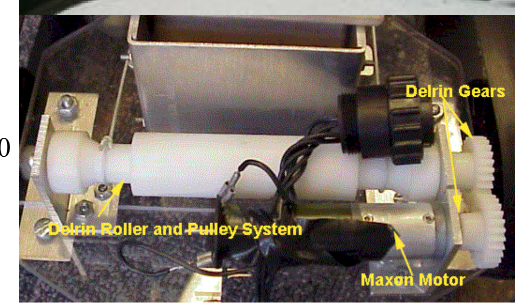












Systems Engineering: *"Optimal" Transmission Ratio*[1]

A transmission can be used to reduce motor speed and increase torque, but what is the optimal (best or very good compromise) transmission ratio? Pasch and Seering[2] did a detailed parametric study of many different factors, and showed that to achieve maximum acceleration of a linear motion system driven by a motor, the optimal transmission ratio *r'* can be found from:

$$\Gamma_{Torque} = \left(J/r + Mr\right)a \qquad \frac{\partial}{\partial r}\left(J/r + Mr\right) = -J/r^2 + M = 0$$

$$r' = n_{optimal} = \sqrt{J/M}$$

This assumes a triangular velocity profile and low frictional and external forces. This is sometimes referred to as the *matched inertia doctrine*: A motor must accelerate its own inertia as well as the inertia of the load. Since both must speed up at the same time, the power to accelerate the motor should equal the power to accelerate the load. For rotary motion systems, the inertia of the gear attached to the motor is added to the motor inertia, and the output gear inertia is added to the load inertia, and the optimal transmission ratio is:

$$n_{optimal} = \sqrt{J_{load}/J_{motor}}$$

For a motor driving a belt or wheels on a car, either directly or through a transmission, the optimal roller or wheel radius can be found in a similar manner:

$$r_{roller} = \sqrt{J_{motor}/m_{load}} \qquad r_{roller} = n_{transmission}\sqrt{J_{motor}/m_{load}}$$

For a motor driving a leadscrew that moves a carriage, the leadscrew inertia is added to that of the motor to arrive at the optimal lead ℓ (distance traveled/revolution of the screw) for the leadscrew:

$$\ell_{(mm)} = 2\pi \times 1000 \times \sqrt{\frac{J_{motor} + J_{leadscrew}}{m_{load}}}$$

When the load inertia is very large, the transmission ratio is often very large, and hence one must always check the motor speed that results from the product of the speed of the load and the transmission ratio. Sometimes the optimal transmission ratio cannot be achieved because the motor becomes too large or its speed becomes too high. In the case where a motor driving a linear motion system reaches its peak velocity, the total travel time t_t and distance d traveled are related by:

$$t_t = \frac{\omega_{max}\left(J + M r^2\right)}{\Gamma_{torque}} + \frac{d}{r\,\omega_{max}} \qquad \frac{\partial t_t}{\partial r} = 0 = \frac{2rM\,\omega_{max}}{\Gamma_{torque}} - \frac{d}{r^2\omega_{max}}$$

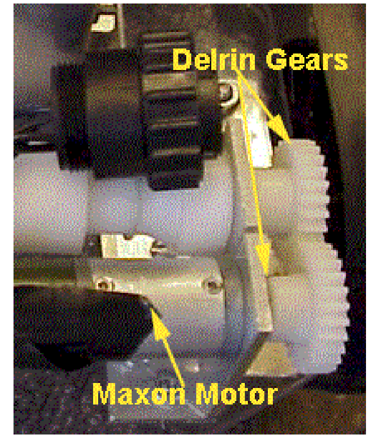
$$r_{optimal} = \sqrt[3]{\frac{\Gamma_{torque}}{2M\,\omega_{max}^2}}$$

For heavily loaded systems, a first order estimate of the optimal transmission ratio can be obtained by dividing the load inertia by the efficiency of the system, or dividing the external force by the acceleration and adding this value to the system inertia. An "exact" method exists which also considers motor thermal issues in the optimization.[3]

What is your optimal wheel size, transmission ratio or lead? Check out motor manufacturers' web sites, as some have nifty optimal drive train design software that will help you select a motor and transmission ratio (e.g., http://www.motionvillage.com/motioneering/index.html). If the load inertia is less than five times the motor inertia, the system will respond acceptably. If the load inertia is greater than ten times the motor inertia, then the system is very likely to have control problems.

1. It is highly advisable to use metric units (mks), else your calculations will proceed with the pace of a slug, and your task is likely to take a fortnight!
2. K.A. Pasch, W.P. Seering, "On the Drive Systems for High Performance Machines", Jou. Mechanisms, Transmissions, and Automation in Design, Mar. 1984, Vol. 106, pp 102-108
3. See A. Slocum _Precision Machine Design_, SME 1992 pp 647-649, or the original reference J. Park and S. Kim, "Optimum speed reduction ratio for d.c. servo drive systems", *Int. J. Mach. Tools Manuf.*, Vol. 29, No. 2, 1989.

Systems Engineering: "*Optimal" Transmission Ratio*

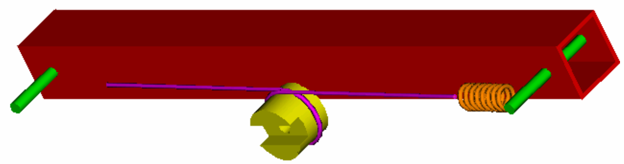


- The "optimal" transmission ratio most efficiently distributes power to the motor/drivetrain and the load, which are connected and must accelerate together
  - Assume mks (meters, kilograms, seconds) units
  - The motor speed ω_{motor} (rpm) to create rotational speed ω_{load} is:

$$\omega_{motor} = \omega_{load}\sqrt{\frac{J_{load}}{J_{motor}}} \Rightarrow n_{\text{transmission ratio}} = \sqrt{\frac{J_{load}}{J_{motor}}}$$

  - For a friction or belt drive system, the motor speed ω_{motor} (rpm) to create linear speed v_{linear} (m/s) is:

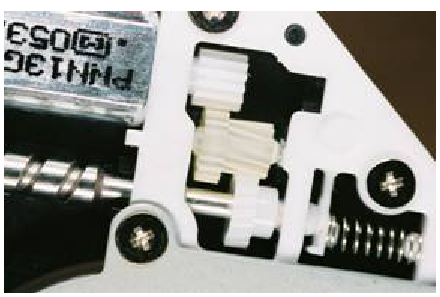
$$\omega_{motor} = \frac{30 v_{load}}{\pi}\sqrt{\frac{m_{load}}{J_{motor}}} \Rightarrow r_{pulley} = \sqrt{\frac{J_{motor}}{m_{load}}}$$



    - The optimal transmission ratio to be placed between a motor and a selected pulley is found by assuming the load inertia is:

$$J_{load} = r_{pulley}^2 m_{load} + J_{roller} \qquad J_{motor} = J_{motor\_rotor} + J_{transmission}$$

  - For a leadscrew driven carriage with lead λ (m traveled/revolution), the motor speed ω_{motor} (rpm) to create linear speed v_{linear} (m/s) is:



$$\omega_{motor} = \frac{60 \times 1000 \times v_{load}}{2\pi}\sqrt{\frac{m_{load}}{J_{motor} + J_{leadscrew}}}$$

$$\lambda(mm) = 1000\sqrt{\frac{J_{motor} + J_{leadscrew}}{m_{load}}}$$

Electricity & Magnetism: *A New Revolution*

Perhaps because electric and magnetic fields are not as easily experienced in nature as are mechanical forces, that the theories required to fully describe and utilize them were not developed until the industrial revolution was well on its way. Perhaps it was the mechanical industrial revolution, with all its marvels and productivity gains, that freed peoples' minds[1] and time to investigate other previously unexplored interesting effects of nature?

Electricity and magnetism are inextricably linked, and the development of fundamental principles of electromagnetism parallels the evolution of modern industrial society. The story can virtually be told with the biographies of several great scientists.[2] Electricity and magnetism were each known separately, but it was Hans Christian Oersted (1771-1851) who demonstrated in 1819 that they are closely related by showing that a compass needle is deflected by a current carrying wire. This inspired many other researchers which led to a flurry of discovery and invention.

André Ampère (1775-1836) was a French mathematician and physicist who built on Oersted's results by showing that the deflection of a compass relative to an electrical current obeyed the right hand rule. Ampère also invented the solenoid, which generates an electric field crucial to so many experiments and devices. Georg Simon Ohm (1789-1854) presented in 1827 what is now known as *Ohm's law:* Voltage (E) equals the products of current (I) and resistance (R) in his now famous book *Die galvanische Kette, mathematisch bearbeitet* which describes his complete theory of electricity[3].

Along with these fundamental discoveries about the nature of electromagnetism, Alessandro Volta, (1745-1827) discovered how to store electrical energy in a battery. Meanwhile, Michael Faraday (1791-1867), an english bookbinder who became interested in electricity, discovered that a suspended magnet would revolve around a current carrying wire. This led to the invention of the *dynamo*, which converted electricity to motion. In 1831 he discovered electromagnetic induction. In 1845 he developed the concept of a field to describe magnetic and electric forces. Faraday, however, was more experimentally oriented, and fortunately, he made his observations widely known.

The mathematically gifted James Clerk Maxwell (1831-1879) was inspired by Faraday's observations to write a paper entitled "On Faraday's Lines of Force" (1856)[4]. He then published "On Physical Lines of Force" (1861) in which he treated E&M lines of force as real entities, based on the movement of iron filings in a magnetic field. He also presented a derivation that light consists of transverse undulations of the same medium which is the cause of electric and magnetic phenomena, which later inspired Einstein. Finally, Maxwell published a purely mathematical theory "On a Dynamical Theory of the Electromagnetic Field" (1865). Maxwell's complete formulation of electricity and magnetism was published in *"A Treatise on Electricity and Magnetism"* (1873), which included the formulas today known as *Maxwell Equations* which are perhaps the most important fundamental relationships ever created.

As it was with the mechanics of solids, it was the invention and development of the calculus by the mathematicians that catalyzed a deep understanding of E&M in a manner that would empower engineers to create ever more complex and powerful devices. Just as Euler and others developed the theory of bending and vibration of beams, development of electromagnetic theory allowed engineers to play "what if" scenarios and to see trends in performance by identifying sensitive parameters. In a world of numerical analysis, where finite element analysis programs generate amazingly beautiful and sometimes extremely useful images, the ability to directly manipulate and use the theory can enable an engineer to synthesize and create like no program ever could. The ability to do the math allows an engineer to create a spreadsheet or a MATLAB script that empowers them to rapidly study parametric relationships and home in on "optimal" designs. Then FEA can be used to check and evolve difficult-to-model details.

1. Speaking of freedom, Benjamin Franklin also helped to launch the Electricity & Magnetism (E&M) revolution with his early curiosity and experiments.
2. For summary biographies of these great scientists, see for example http://scienceworld.wolfram.com/biography/
3. http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Ohm.html
4. "…Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance. Faraday saw a medium where they saw nothing but distance. Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids."
James Clerk Maxwell [1873]

Electricity & Magnetism: *A New Revolution*

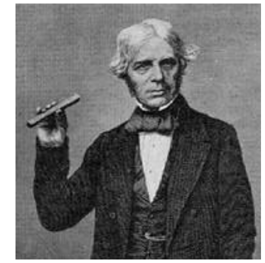
Georg Ohm (1789-1854)

Hans Oersted (1771-1851)

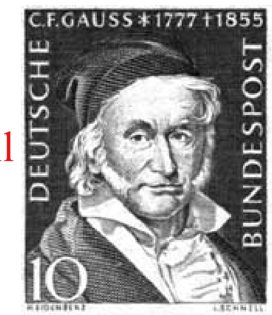
Gustav Kirchoff (1824-1887)

- Ohm's law:
  - Voltage (*electromotive force*) in a circuit = *current* x *resistance* (E = IR)
  - The *magnetomotive force* F_m (*MMF*) in a magnetic circuit is proportional to the *magnetic flux* Φ (*flux*) and the *reluctance* R ($F_m = \Phi R$)
- Kirchoff's current and voltage laws:
  - The sum of all currents (*fluxes*) flowing into a node is zero
  - The sum of all voltage drops (*MMF*) in a closed loop equals zero
- Faraday's law of electromagnetic induction:
  - Coils of wire and magnets interact to create electric and magnetic fields
- Ampere's law:
  - An "electromotive force", such as created by current passing through a coil of wire, *forces* a magnetic field through a magnetic circuit
- Gauss's law for magnetism:
  - Magnetic fields have North & South poles between which the field flows

André Ampère (1775-1836)



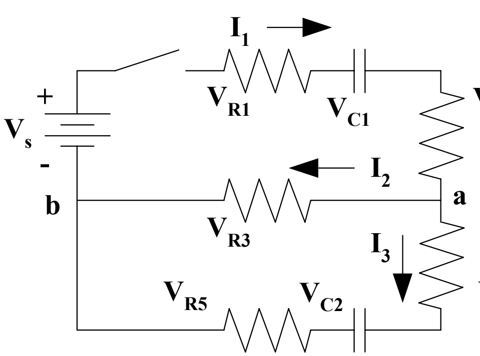
Michael Faraday (1791-1867)



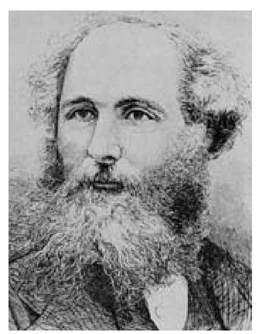
Johann Carl Friedrich Gauss (1777-1855)

James Clerk Maxwell (1831-1879)

And then there was (an understanding of) light!



$$V_S = V_{R1} + V_{C1} + V_{R2} + V_{R3}$$
$$0 = V_{R3} + V_{R4} + V_{C2} + V_{R5}$$
$$I_1 = I_2 + I_3$$



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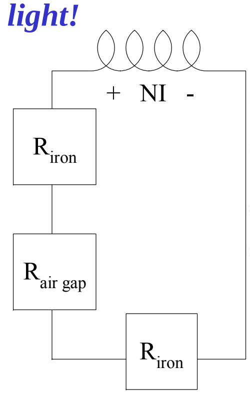
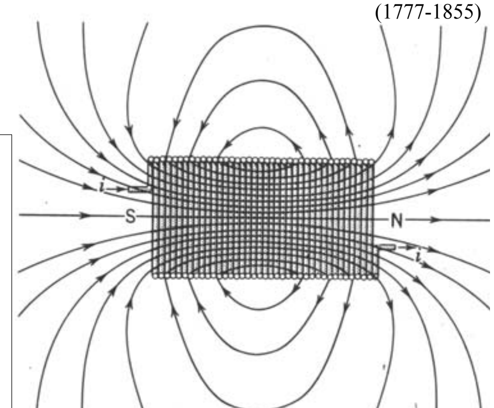




From D. Halliday & R. Resnick, *Physics Parts I & II Combined 3rd edition*

Electricity & Magnetism: *FUNdaMENTAL Principles*

Electric and magnetic fields are analyzed by defining a boundary and then applying one of the fundamental principles (e.g., Faraday's, Ampere's, or Gauss's laws) to that boundary. Given an electric or magnetic field that crosses a closed boundary (or surface), Faraday's, Ampere's, & Gauss's laws essentially say that the sum of all of the products of the infinitesimal components of a field with all of the infinitesimal lengths (or areas) of a closed boundary are equal to some scaler value. Independent of the complexity of the field or closed boundary, they are expressed in a most general form as surface integrals of the dot products of field and boundary vectors.

Real devices can often be modelled with a two-dimensional boundary and the multi-variable calculus problem becomes a simple summation of scaler quantities. An example is the application of Ampere's Law to the force produced by an electromagnet, where a complete magnetic circuit can be analyzed as the sum of the products of the magnetic field intensities *H* aligned with the path and perpendicular to cross sectional areas *A*.

Table 1:

Electric Circuit	Magnetic Circuit
E (Volts) electromotive force (EMF)	F_m = NI (At=NI=ampere-turns) magnetomotive force (MMF) H = F_m/L_{length} (At/m = Oersteds) magnetic field intensity
I, *i* (Amperes) current	$\Phi = F_m/R_m$ (Wb = Webers) Magnetic flux B = Φ/A_{area} (Wb/m^2=N/(ampere-meter) = T = Tesla) magnetic induction or flux density
$\rho = 1/\sigma$ (conductivity)	μ = B/H (Henries/meter = H/m = tesla-meter/ampere) permeability
R (Ohms) resistance	R_m = L/μA (At/Wb) Reluctance

When applying the fundamental laws to magnetic (or electric) circuits, the magnetic flux (or current) through each components in series is equal (they form a voltage divider), and the magnetomotive force (or voltage) through components in parallel is equal (they form a current divider). Gustav Robert Kirchoff (1824-1887) wrote these laws of closed electric circuits in 1845, which are now known as Kirchoff's Current and Voltage Laws: *The sum of voltage drops around a circuit will be equal to the voltage drop for the entire circuit.* In fact, the laws for magnetic and electric circuits (and also for fluid circuits!) are similar as shown in Table 1, and by Ohm's law:

$$E = IR \qquad N_{\#\text{ of turns}} \times I_{\text{current}} = F_m = \Phi R_m = B A_{area} R_m$$

Consider the force of attraction between an electromagnet and an object[1]. For the figure shown, it can be assumed that the reluctance in the magnetic circuit is dominated by the air (permeability μ_o) in the region with gap δ and area *A*. Applying Ampere's law yields:

$$H \times \delta = N \times i \Rightarrow H = \frac{Ni}{\delta}$$

$$H = \frac{B}{\mu} \Rightarrow B = \frac{\mu_0 Ni}{\delta} \qquad B = \frac{\Phi}{A} \Rightarrow \Phi = \frac{\mu_0 NiA}{\delta}$$

The magnetic flux Φ also passes through the center of each turn of the coil. Faraday's law says that this will induce a voltage in each turn of the coil. Since each turn is linked in series, the total magnetic flux linked together by the coils is $\lambda = N\Phi$, where λ is called the *flux linkage*. The definition of inductance *L* is $\lambda = Li$, and since inductance in an electrical system is like mass in a mechanical system, and current is like velocity, the energy (work) *U* and hence the attraction force are:

$$\lambda = N\Phi = \frac{\mu_0 N^2 iA}{\delta} \qquad \lambda = Li \Rightarrow L = \frac{\mu_0 N^2 A}{\delta}$$

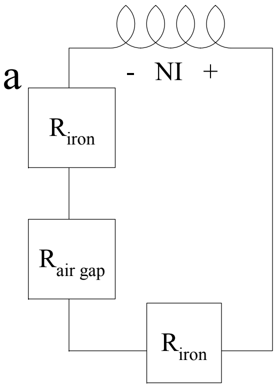
$$U = \frac{1}{2}Li^2 \qquad F = \frac{\partial U}{\partial \delta} = -\frac{\mu_0 N^2 A i^2}{2\delta^2} = -\frac{B^2 A}{2\mu_0}$$

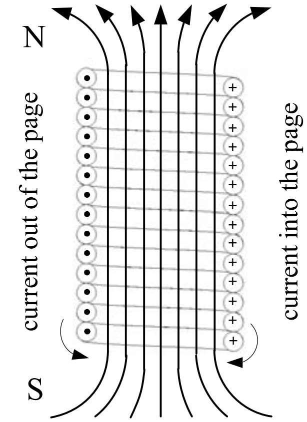
Now would be a good time to review your electricity and magnetism notes and text from your freshman physics course!

1. Many thanks to Prof. Jeff Lang for providing this clear explanation of a simplified system. For an in-depth discussion of this and other related topics, see Electromechanical Dynamics, HH Woodson and JR Melcher, John Wiley & Sons, 1968, Volumes I & II

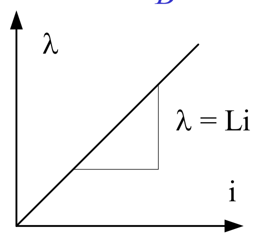
Electricity & Magnetism: *FUN*da*MENTAL Principles*

- Faraday's law of electromagnetic induction:
  - A force F is required to move a conductor of length L carrying a current i through a magnetic field of strength B ($F = BLi$)
  - A magnet that moves in a coil causes a potential difference at the terminals of the coil
    - Current that flows through a coil, creates a magnetic field
      - The time varying change in a magnetic field Φ_B induces an electromotive force E:

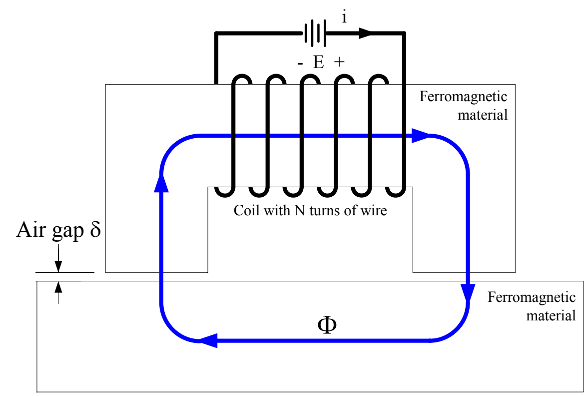




$$\oint \mathbf{E} \bullet d\mathbf{l} = -\frac{d\mathbf{\Phi_B}}{dt}$$

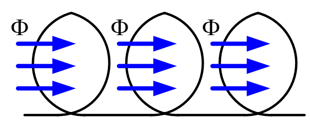


Definition of inductance L

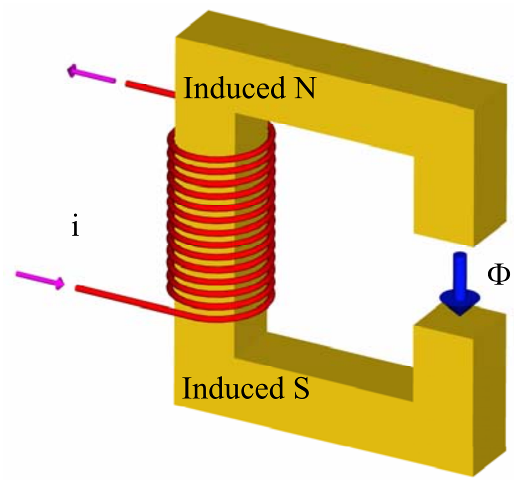


- Ampere's law:
  - The product of the magnetic field intensity H and length in a circuit equals the magnetomotive force:

$$\oint \mathbf{H} \bullet d\mathbf{l} = Ni = F_m$$



Flux linkage $\lambda = N\Phi$: N turns of a coil, in series with each other and linked together by magnetic flux Φ



- Gauss's law for magnetism:
  - The total magnetic flux ($\Phi = B * A_{area}$) across a closed boundary is zero (there are no magnetic monopoles):

$$\oint \mathbf{B} \bullet d\mathbf{S} = 0$$

Magnetic Circuits

Permanent magnets and electromagnets come in many shapes, sizes and forms, based on the strength and shape of the desired magnetic field. In most cases, they are used to "pull" or attract an object; hence they can be used to preload a system of bearings[1] or to pick up magnetic materials. When designing a system with magnets it is important to keep in mind the fundamental principles of magnetic circuits and to apply Ampere's and Gauss's laws: *Magnetic systems require a closed circuit for the field to flow, just the way an electric system requires a closed circuit.*

Resistance to flow, the *reluctance R*, of magnetic flux Φ is low in magnetic materials with high *permeability* μ. Examples include ferrous (iron and steel) materials, cobalt and nickel. However, the permeability is a function of the strength of the *magnetic field intensity* (strength) H (units of ampere-turns/meter or Oersteds) and the *magnetic induction* (flux density) B (units of tesla = 10,000 gauss). H is a measure of the ability of an electric current or a magnetic body to induce a magnetic field at a given point. The plot shows some typical values for various materials. As the flux density increases, the field intensity increase slows as the material saturates with magnetic field, and the permeability drops. This is why magnets themselves have such low permeabilities. The permeability of non-magnetic materials, including other metals, air, and insulators, can be considered equal to free-space $\mu_o = 4\pi \times 10^{-7}$ *H/m*. Hence most magnetic circuits' performance are limited by air gaps.

As was shown, in order for a magnet to attract an object made from magnetic materials, a certain magnetic flux density B must be established across the air gap, and hence a required magnetic flux $\Phi = BA$ must flow through the magnetic circuit. However, the flux Φ will be resisted by the reluctance of all the elements in the magnetic circuit. If the reluctance is high, such as due to a large air gap, a large amount of *MagnetoMotive Force* F_m (MMF) will be required to "push" the flux through the circuit. To determine how strong a permanent magnet or an electromagnet must be, first calculate the magnetic induction (flux density) B (Tesla) across the airgap that is required to create the desired force. The next step is to determine the total reluctance of the magnetic circuit. The MMF (F_m) required to yield the needed flux density can then be determined from Ohm's law for magnetic circuits: For magnetic circuits with elements having path lengths L and areas A with permeabilities μ:

$$F_m = F \times R = \frac{F \times L}{A} = \frac{H \times A \times L}{A} = H \times L = \frac{B \times L}{\mu}$$

Magnetic lifting systems are therefore straightforward to design, although for any production system, finite element analysis would be used to optimize the system. Even so, a first-order analysis is highly recommended as a sanity check for any FEA, and this includes determining:

1) Flux density $B_{required}$ to create the desired force: $B_{required} = \sqrt{\dfrac{2\mu_0 F_{\text{desired}}}{A_{\text{Area air gap}}}}$

2) Flux $\Phi_{Circuit}$ in the magnetic circuit: $\Phi_{Circuit} = B_{required} A_{\text{Area air gap}}$

3) Flux density B_j in each circuit element j: $B_j = \dfrac{\Phi_{Circuit}}{A_j}$

4) Permeability μ_j for each circuit element j from material BH curves,

5) Reluctance R_j for each circuit element j: $R_j = \dfrac{L}{\mu_j A_j}$

4) Magnetomotive force: $F_{m\ \text{total}} = \Phi \sum_j R_j$

5) Ampere-turns NI: $NI = F_{m\ \text{total}}$

The total magnetomotive force $F_{m\ total}$, is the "strength" of the magnet required in Ampere turns. If the magnet is made by wrapping wire around a metal section of the circuit, then the product of the current I and number of turns of wire N must be: $NI = F_{m\ total}$. If a bar magnet is to be placed in the metal yoke (flux focussing device), then $F_m = L_{magnet\ length} \times Strength_{Oersteds}$. Note the permeability μ of magnets is very low, close to that of air, because they are already saturated.

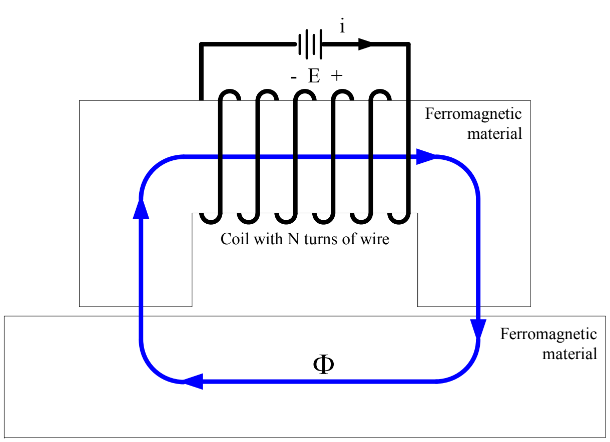
Try designing a simple electromagnet and see if it can lift the predicted load with the predicted power. If you design a bar-magnet lifting device, how can you make it let go of the load?

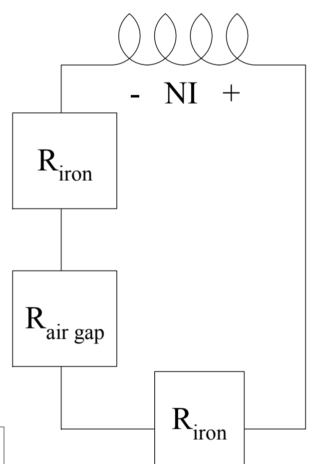
1. See the discussion on the Axtrusion Design on page 1-15

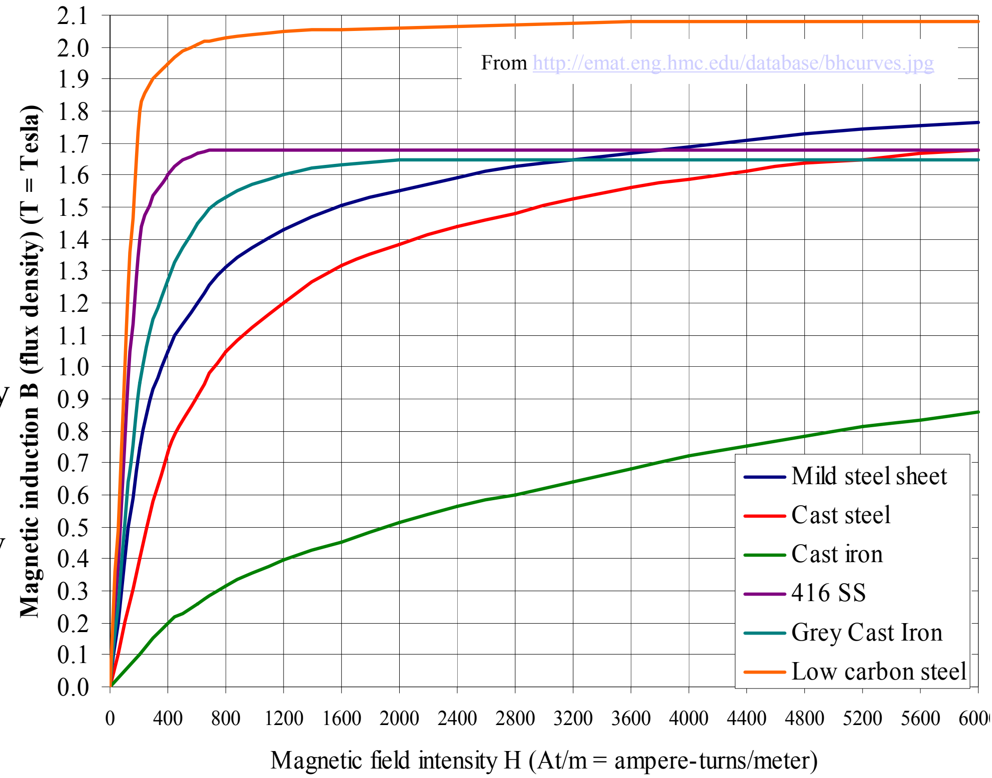
Magnetic Circuits

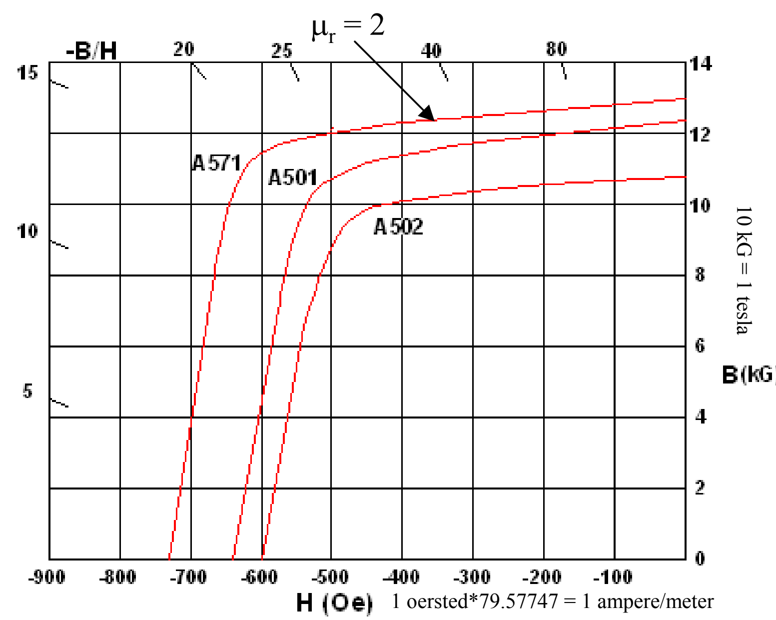
- Magnetic flux Φ flows in a circuit, just like electricity:
  - Permeability $\mu = B/H$: ability of a magnetic field to permeate (flow) through a material
  - $\mu_0 = 4\pi \times 10^{-7}$ (tesla-meter/ampere) = permeability of free space and non-magnetic materials
  - $\mu_r = \mu / \mu_0$ = relative permeability
  - Reluctance R is a measure of the resistance to flow of magnetic flux through a particular piece of material:
    $$R = \frac{L_{\text{total path length in the material}}}{\mu A_{\text{cross sectional area along the path}}}$$
  - Attraction force F_{MT} across an air gap:

$$F_{MT} = \frac{B^2 A}{2\mu_o} = \frac{\Phi^2}{2\mu_o A}$$









Alnico A501 from http://www.mceproducts.com/materials/Alnico_Details.asp?id=1047

Magnetic Circuits: *Permanent Magnets*

Manufacturers of permanent magnets provide B-H curves such as those shown for a common low-cost aluminum nickel cobalt (AlNiCo) and more expensive and stronger Neodymium Iron Boron (NdFeB) and Samarium Cobalt magnets. Notice that the H axis is negative because the magnet is a source. In addition to the B-H curves for a magnet, other specified parameters include:

- *Curie temperature*: The transition temperature above which a material loses its magnet properties.
- *Maximum operating temperature*: The temperature at which the magnet can be used without significant reduction in strength. Typically, $\Delta B_r/^oC$ = -0.02% for AlNiCo, $\Delta B_r/^oC$ = -0.12% for NdFeB, $\Delta B_r/^oC$ = -0.05% for SmCo magnets
- *Remanence*: The magnetic induction, B_r, that remains in a magnetic material after removal of an applied saturating magnetic field used to initially magnetize the magnet.
- *Equivalent ampere turns*: For a permanent magnet:

$$(N \times i)_{\text{equivelent}} = \frac{-B_r L_{\text{magnet length}}}{\mu_{\text{magnet material}}}$$

Permanent magnets' non-linear B-H curves' kink indicate *demagnetization*: For example, when a brushless DC motor is manufactured, the rotor is placed into the stator, which contains the magnets, and a very strong external magnetic field is applied to the assembly. The system then operates along the closed circuit load line. However, if the iron rotor is removed from the motor, the magnetic circuit experiences the open circuit load line and the B-H curve is intersected beyond the kink. The red-dashed line is the *recoil line*, that represents the new BH curve for the magnets which have become demagnetized and have lost some strength, and as a result, the motor has a lower torque constant because the windings generate a magnetic field that opposes the motor's permanent magnets. This same demagnetization effect can happen to a motor when too high a current is supplied to the windings.

A purchased permanent magnet has a maximum B_r value, and it must be held in a flux focussing housing so the resulting magnetic circuit does not result in a load line that demagnetizes the magnet when the system is in an open magnetic circuit. For example, the closed circuit load line with the red-dashed recoil line defines the state for the system when it is attached to an object. When the magnet system is pulled away, the open circuit load line comes into play, but it intersects the recoil line and the magnet operates in this region. This is why some magnets are shipped with an iron keeper bar.

Simple magnetic circuits, where the air gaps are very small compared to magnet dimensions, can be analyzed using first principles. For greater accuracy or for more complex geometries, many programs available online[1]. Assume a magnet has length L_m and cross sectional area A_m. Make sure that the magnet's remanence (magnetic induction B_r) is twice that of the entire flux density $B_{required}$ required to generate a lifting force as found above. To find the load line, the reluctance through the magnet should equal the reluctance $R_{external}$ through the rest of the circuit (iron and air gap). Because the flux Φ through the external circuit equals that through the magnet:

$$F_m = \Phi R \Rightarrow B_m A_m R_{external} = -H_m L_m \Rightarrow \frac{B_m}{H_m} = \frac{-L_m}{A_m R_{external}}$$

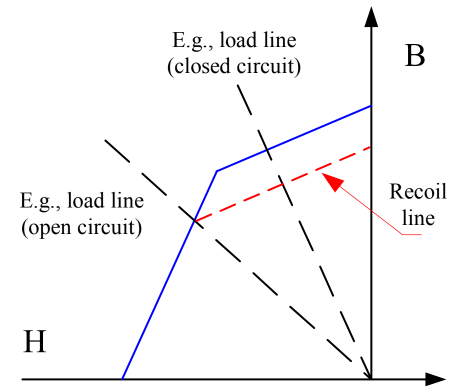
Permanent magnets are very brittle, and can be cut by grinding, EDM or an abrasive waterjet. In general, magnets should be ordered to size. Some magnets are made by mixing powdered magnetic material with a polymer and then formed by injection molding or extrusion. The resulting shape (e.g., refrigerator magnets) is then magnetized. Alternatively, adhesive-backed sheet-magnets of substantial magnetic strength, due to their large areas, can be purchased, cut, and stuck to objects. For example, a robot design contest control box with a sheet-magnet bonded to it can be rapidly, securely, and removably attached to the sheet-metal body of a robot.

Can permanent magnets be used as holding devices for a spring-loaded system, where they hold most of the force of a spring, and then just a little bit of externally applied force from a trigger releases the system? How else can you use permanent magnets in your design?

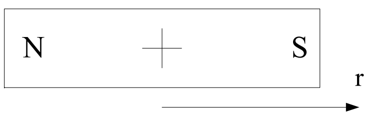
---

1. *Permanant_magnet.xls* can be used to help design simple settings. For precise field calculations, see http://www.mceproducts.com/welcome.asp and the link to http://www.mceproducts.com/knowledge/knowledgedt.asp?id=66 Note that 1 gauss = 10^{-4} tesla, and 1 maxwell = 10^{-8} weber

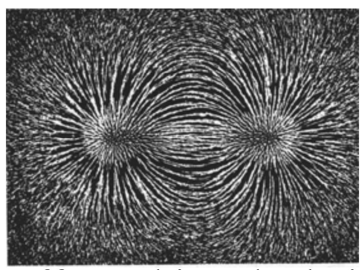
Magnetic Circuits: *Permanent Magnets*



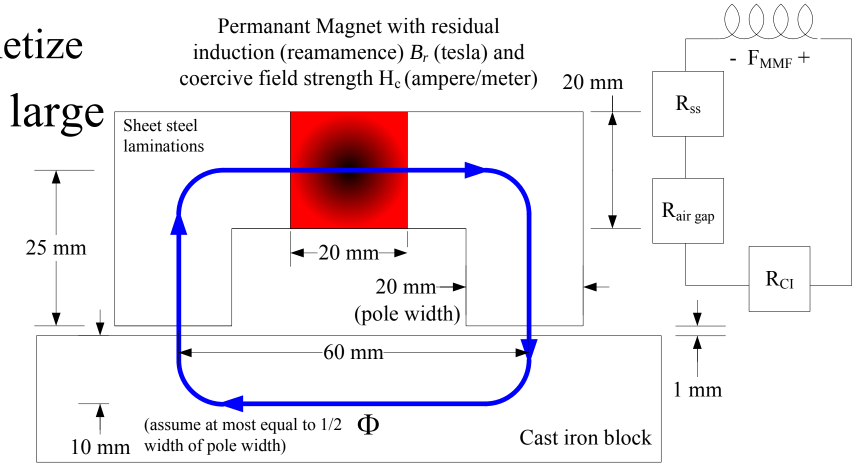
- Permanent magnets provide magnetic flux which flows from their *North* pole to their *South* pole
  - In a circuit, they act as a "voltage" source, $MMF = F_m = BL/\mu_o$
  - The "current" (flux density), $B = \Phi/A$, associated with the "voltage" depends on the rest of the circuit
  - If too much "current" is drawn, they will demagnetize
- Flux density in space of a dipole along its axis a large distance from its center (NiA=Ampere-meter²):



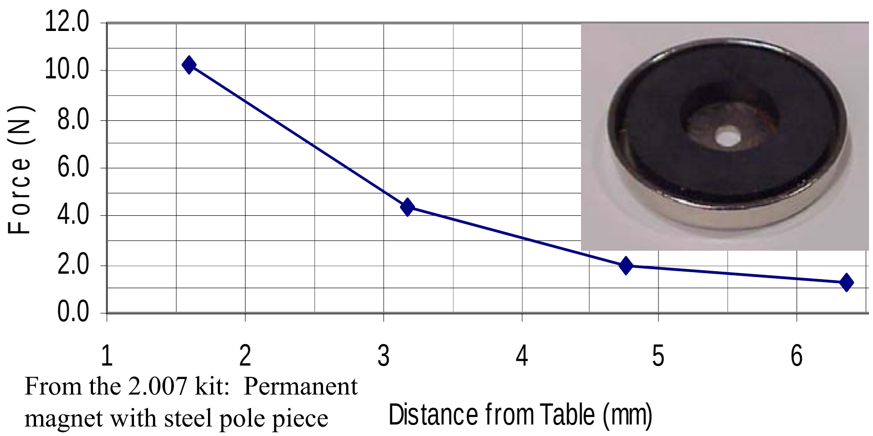
$$B(tesla) = \frac{\mu_o NiA}{2\pi r^3}$$



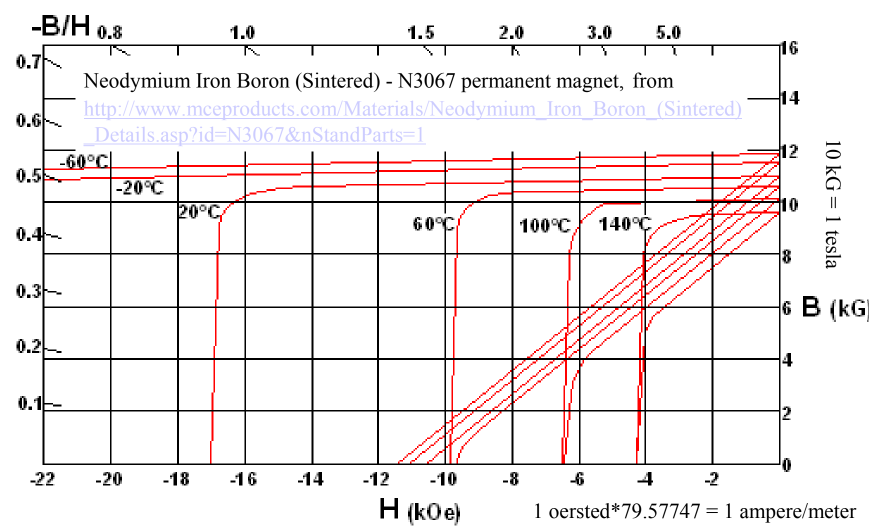
From D. Halliday & R. Resnick, *Physics Parts I & II Combined 3rd edition*



  - Magnet geometry affects this calculation (A LOT! See http://www.mceproducts.com/)



From the 2.007 kit: Permanent magnet with steel pole piece



Magnetic Circuits: *Applications*

Several applications of magnets are shown here, including a common lifting application that uses an electromagnet; and wheels that use a thin round permanent magnet bonded between two round steel plates, which enable them to be attracted to a steel surface.

Electromagnets are easy to design and build, and can be turned on and off, as shown in the spreadsheet *Magnet_Lifting.xls* which performs the calculations described earlier for the figure shown. A portion of the spreadsheet is shown below:

Lifting_electromagnet.xls	
By Alex Slocum	
Last modified 6/22/03 by Alex Slocum	
To determine magnet properties to lift a mass, remember, like current in a series circuit, the flux in a series magnetic circuit is constant.	
Enters numbers in **BOLD,** Results in **RED**	
Mass of object to lift, M (kg)	**0.5**
Force required to lift object, F (N)	**4.9**
Total required Magnetomotive force, MMFtotal (At)	**37.6**
Available current, Iavail (amps)	**0.5**
Minimum number of required turns	**75.2**
Air gap	
Area of first pole, Ap_1 (mm^2)	**600**
Length of first air gap, Lag1 (mm)	**0.1**
Area of second pole, Ap_2 (mm^2)	**600**
Length of second air gap, Lag2 (mm)	**0.1**
Required magnetic flux density to lift object, Breq (Tesla)	**0.101**
Magnetic field intensity Fm @Breq, MMFag (At)	**16.1**
Required magnetic circuit flux, phi (Wb)	**6.08E-05**
Lifting magnet	**sheet steel**
Section 1	
Magnetic circuit path length, L_1 (mm)	**25**
Magnetic circuit path area, A_1 (mm^2)	**600**
Flux density, B_1 (Tesla)	**0.101**
From B-H curve, magnetic field intensity, H_1 (At/m)	**50**

Permanent magnets could also be used to pick up a load, but then how could the load be dropped off? How is a magnetic base for a dial indicator in the shop attracted to an iron surface, but then with the flick of a lever it is released? Think of the electrical circuit analogy: Could the lever introduce a parallel lower reluctance path into the circuit, thereby directing most of the flux into a different path? Would it be better to break the magnetic circuit with a switch? Could it be done? How feasible would this be compared to making an electromagnet?

There are other applications for permanent magnets besides lifting objects. The most common is probably in permanent magnet DC motors, which are discussed later in this chapter. Other interesting applications include using them to preload components so they are forced together. For example, consider the robot contest table with the metal ramp for MIT's 1999 *Mechever-est*. Several students realized that it was impossible to drive up the 45 degree slope using just gravity, so they made magnetic wheels by sandwiching a permanent magnet between two steel disks. The concept of magnetic wheels that are attracted to a metal surface is decades old, and if the students had done a little bit of searching on the web, they could have had a much better wheel design (recall page 2-14 and the "References" column of the FRDPARRC sheets!). Later, another group of students used this principle for their *Battlebot*, the *General Gau*[1]. This same principle was used for the *Magnabots*[2], which are designed to be attracted to steel plates attached to ceilings and walls for automation in environments such as hospitals. In this application, light loads need to be frequently transported, and the hallways are too crowded with people to allow mobile robots to efficiently roam.

Study the pictures and plots provided for magnets, and play with the spreadsheet. Try designing an electromagnetic magnetic lifting circuit and then build and test it. Will it lift the desired weight? Could your estimation of the air gap affect your prediction? What is the most sensitive variable in the analysis?

---

1. The General Gau had unbeatable traction, and in its first contest, it effortlessly pushed its opponent back with such speed, that when the robot hit the wall, the opponent's 'bot flew into pieces. Unfortunately, the brilliant designers of the General Gau did not tie down their internal electronics which also broke free and killed the General Gau. The opponent, meanwhile, could still twitch and hence won the match! Details, details, details...many a great idea has died for want of details!
2. Slocum A., Awtar S., Hart J., "Magnabots: A Magnetic Wheels Based Overhead Transportation Concept", Proceedings of the 2nd IFAC Mechatronics Conference, 2002, p. 833. See http://pergatory.mit.edu/magnabots

Magnetic Circuits: *Applications*

- Magnets can be used for lifting objects:
  - Permanent magnets will attract an object, but how do you get them to let go?
  - Electromagnets are easy to design and use, and can let go of objects
- Magnets can be used as wheels on metal surfaces:
  - First used for the 1999 2.007 contest Ballcano
  - See http://pergatory.mit.edu/magnebots/

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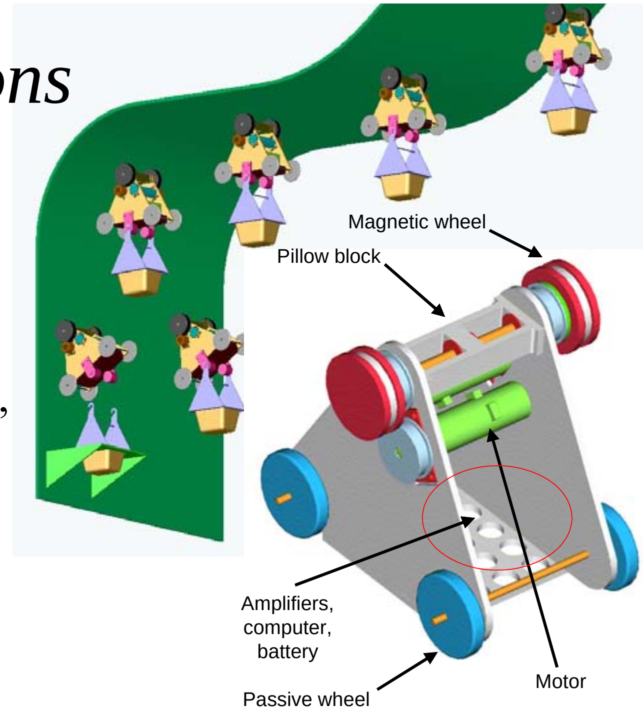
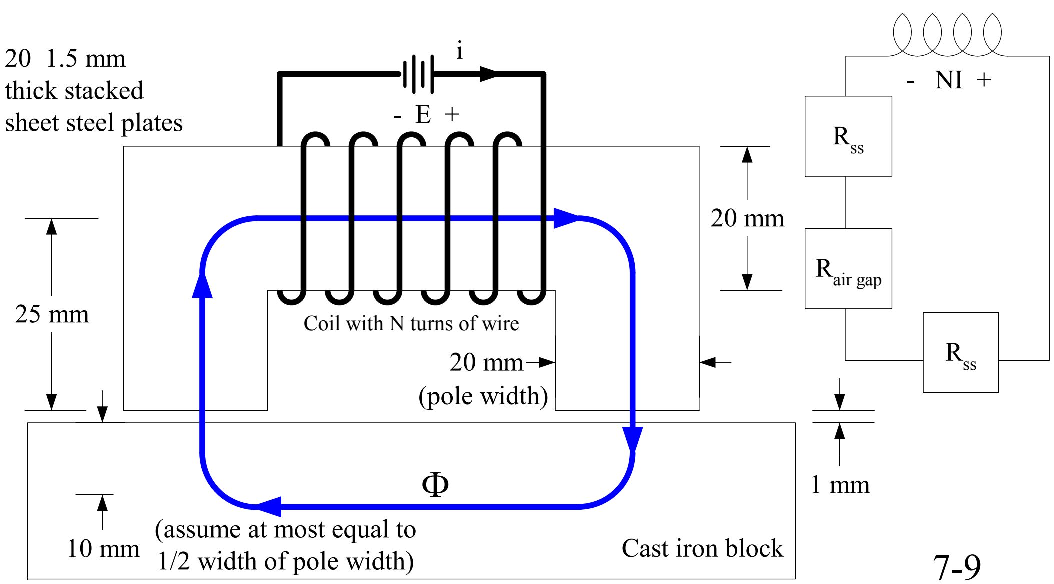
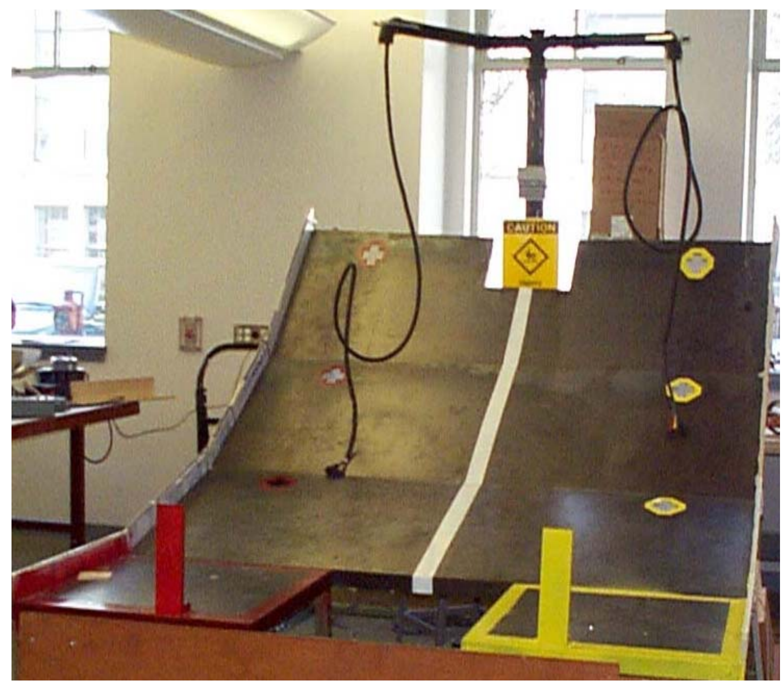






Magnetic Circuits: *Permanent Magnet Attraction*

Consider using a permanent magnet as a lifting device. How does one use the theory presented earlier and create a spreadsheet to determine how strong a force can be generated? An effective method is to literally write down the flow of thought, and then embody it into the spreadsheet:

The magnetomotive force (MMF) produced by a magnet of strength *B* (tesla) and thickness *L* (m) is like a voltage source in a circuit, where the permeativity of the magnet is about equal to that of free space because it is saturated:

$$F_{\text{MMF magnet}} = \frac{BL}{\mu_0}$$

The reluctance (resistance to flow of magnetic flux) of the magnetic circuit that resists the magnetic force is comprised of the reluctances of the magnet itself, the flux focusing ferromagnetic elements of the device, the air gap, and the ferromagnetic material being attracted. The reluctance in any of these elements is a function of the path length *L* through the material, the cross sectional area *A* normal to the path length, and the permeativity of the material:

$$R = \frac{L}{\mu A}$$

For air, $\mu = \mu_0$ and L = the air gap. Assume that the ferromagnetic materials in the circuit are not saturated, and the magnetic flux density *B* (tesla) is equal to that of the magnet. Given an assumed *B*, *BH* curves for different materials can then be used to find μ for each material in the gap.

Once all the reluctances have been determined, they are added to find the total reluctance R_{total} for the circuit. The magnetic flux that flows through the circuit can then be found:

$$\Phi_{\text{circuit}} = \frac{F_{\text{MMF magnet}}}{R_{\text{total}}}$$

The force that the system can generate on the ferromagnetic material being attracted is then given by:

$$F_{\text{attraction}} = \frac{\Phi_{circuit}^2}{2\mu_0 A_{\text{total air gap area}}}$$

As shown in the spreadsheet *Lifting_permanent_magnet.xls*, several assumptions are made regarding equal cross sections; however, with well-annotated cells, it would be easy to modify the spreadsheet.

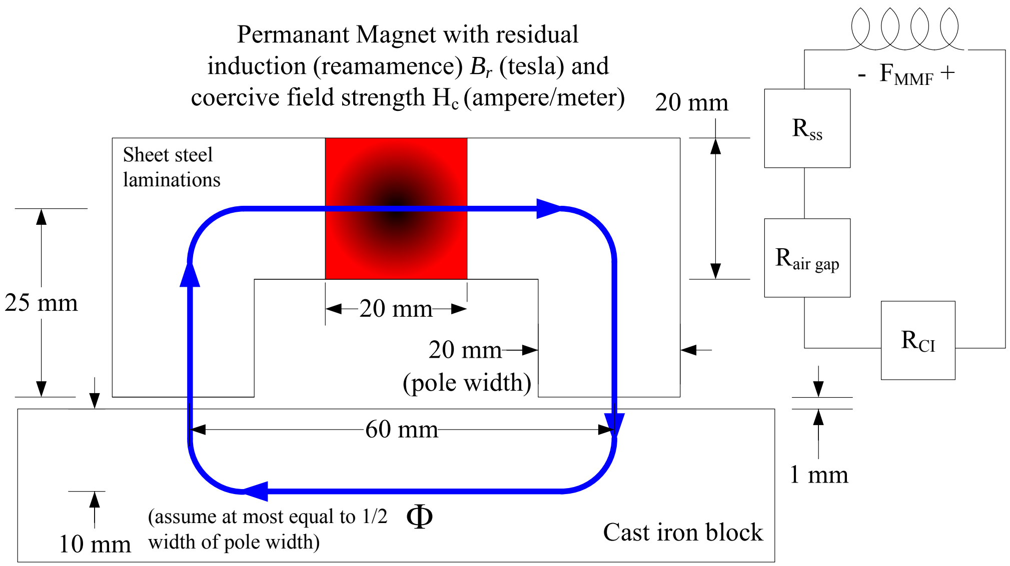
Note the effective magnetic pressure value. An “OK” system can achieve 0.2 atm of attraction pressure[1]. A very good system can achieve 0.5 atm. Designing with permanent magnets can be fun, much like designing circuits. In fact, its not difficult to realize a permanent magnet attraction!

Look at the cells of the spreadsheet and determine the most critical parameters. In general, the air gap is the most sensitive parameter, but if the magnet is too thick, then its own reluctance can be a limiting factor. Is this why magnets are generally thin compared to their other dimensions? Examine the other parameters. Is the cast iron saturating? Will this require the magnetic field strength of 1 Tesla to be reduced? Does it make sense to buy stronger or weaker magnets for this design? What might be the value of more in-depth analysis, using finite element analysis for example, versus building and testing a bench level experiment?

1. 1 atm = 100,000 N/m^2 (Pa) = 0.1 N/mm^2 = 14.7 lbf/in^2 (psi)

Magnetic Circuits:
Permanent Magnet Attraction

- Permanent magnets can be used to provide all or most of the lifting force in a system
- To turn off the force, a "shunt" can be moved into place to provide a low reluctance path for the flux so it does not permeate through the air gap.



Lifting_permananet_magnet.xls	
By Alex Slocum, Last modified 10/5/04 by Alex Slocum	
Enters numbers in **BOLD,** Results in **RED**	
Mass of object to lift, M (kg)	**0.5**
Force required to lift object, F (N)	**4.9**
Results	
Total circuit reluctance, Rtotal	**60808361.9**
Total magnetic flux in the circuit, phi	**2.62E-04**
Maximum attractive force, Fmax (N)	**22.7**
Effective magnetic "pressure" (atm)	**0.2**
Safety Factor	**4.6**
Magnet	
Strength of magnet, Bmagnet (tesla)	**1.00**
Length of Magnet, Lmag (mm)	**20**
Width, wmag (mm)	**20**
Depth (into page) dmag (mm)	**30**
Magnetic circuit path area, Amag (mm^2)	**600**
Reluctance (resistance), Rmag	**59683103.7**
Magnetomagnetic force, Fmmf_magnet (At)	**15915**
Flux Focussing Metal (values per side unless noted)	**Sheet steel**
Magnetic circuit path length, L_1 (mm)	**45**
Width (at pole), w_1 (mm)	**20**
Depth (into page) d_1 (mm)	**30**
Magnetic circuit path area, A_1 (mm^2)	**600**
Flux density (assume equal to magnet), B_1 (Tesla)	**1.00**
From B-H curve, magnetic field intensity, H_1 (At/m)	**400**
Reluctance per side	**30000**
Total flux focussing metal reluctance, Rss	**60000**
Air gap (assume 2 sides equal, values per side unless noted)	
Air gap, Lag1 (mm)	**0.1**
Air gap reluctance	**132629**
Total air gap reluctance, Rair_gap	**265258**
Object being lifted	**cast iron**
Magnetic circuit path length, Lobl (mm)	**80**
Width, wobl (mm)	**20**
Depth (into page) dobl (mm)	**30**
Magnetic circuit path area, Aobl (mm^2)	**600**
Flux density (assume equal to magnet), Bobl (Tesla)	**1.00**
From B-H curve, magnetic field intensity @Breq, Hobl (At/m)	**6000**
Reluctance (resistance), Robl	**800000**
Permeativity of free space, mu0 (H/m)	**1.26E-06**

Magnetic Circuits: *Solenoids*

A solenoid is a long current-carrying wire wound in a close-packed helical structure. The magnetic fields generated by the wires cancel each other in the region between the wires, and reinforce each other on the inside of the structure. The result is the magnetic field shown, and this field will exert a force on an iron plunger in the center of the solenoid. A solenoid is thus essentially a form of an electromagnet, and indeed the calculations are very similar to those used to predict the lifting capability of an electromagnet. Estimating the force that can be produced by a solenoid is not too difficult. Finite element analysis can also be used with great accuracy. However, a design engineer most often selects a solenoid out of a catalog and just has to be aware that the force typically drops off very quickly with stroke of the plunger.

Hans Oersted (1771-1851) is credited with making the discovery that a current carrying wire produces a magnetic field. André Ampère (1775-1836) discovered the relation (Ampère's law) that describes the magnetic effect in free space caused by a current in a wire[1]

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o i$$

For a circular path around a wire, the angle between the magnetic field **B** and the path length vector d**l** (which is tangent to the contour of interest) is 0, so **B**·d**l** = Bdl. Faraday's law describes the effect of a changing magnetic field on the current flowing in a wire. The magnetic field strength B in *air* in the middle of a solenoid is created by M turns/meter $(M = 1/D_{wire})$ with n layers carrying a current i; and the magnetic flux Φ_B is the product of the field strength B and the cross-sectional area A of the solenoid of diameter D:

$$B = \mu_o iMn \qquad \Phi_B = B\pi D^2/4$$

When an iron rod (the solenoid's plunger) is placed in the solenoid's magnetic field, a magnetic field is induced in the rod. The system wants to minimize its potential energy, which means it wants to close the gap between the end of the plunger and the bottom of the solenoid casing. This creates a force on the plunger. When the end of the plunger and the bottom of the solenoid casing touch, the fields are aligned, the reluctance (resistance to flow of the magnetic fields) is minimized, and the pull-force on the plunger is maximized. This type of actuator is referred to as a *reluctance force* actuator (or *reluctance torque* for rotary systems). To estimate the force exerted on the rod, the magnetomotive force $F_M = N_{turns} I_{current} n_{layers}$ created by the solenoid coils is found by first determining the reluctance of each element in the magnetic circuit. This enables the determination of the total circuit flux Φ. With Φ, the field strength B in the air gap between the plunger end and body is found from $B_{plunger} = \Phi/A_{plunger}$ and then the force F can be determined. These steps are carried out in the spreadsheet *solenoid_force.xls*.[2]

1. $\mu_o = 4\pi x 10^{-7}$ tesla-meter/ampere = permeability of free space and non-magnetic materials.

In order to maximize the force capability of a solenoid in a given area, the number of turns and/or the current can be increased. Unfortunately, as the wire diameter is decreased to increase the number of turns, its current carrying capability also decreases. The electrical power dissipated as heat in a copper wire with resistivity C (ohm-mm), length L_{wire} (mm) and diameter D (mm) carrying a current i and of a coil of length L_{coil} made from this wire are:

$$P_{\text{dissipated}} = i^2\left(\frac{4CL_{\text{wire}}}{\pi D^2_{\text{wire}}}\right) \qquad P_{\text{dissipated}} = i^2\left(\frac{4Cn_{\text{wound layers}}L_{\text{coil}}L_{\text{wire length per turn}}}{\pi D^2_{\text{wire}}}\right)$$

Current is "expensive", because it causes elements to get hot. Thus coils are often designed with a small wire diameter and more turns. To keep the wire from getting hot to the point where it burns through the wire's insulation and causes a short, the coil must be attached to a structure which acts as a heat sink. The spreadsheet *wire_resistivity.xls* can help determine how much power is needed to energize a coil. Resistance in a copper wire increases by 0.4%/^{o}K, so as it gets hotter it will dissipate even more power!

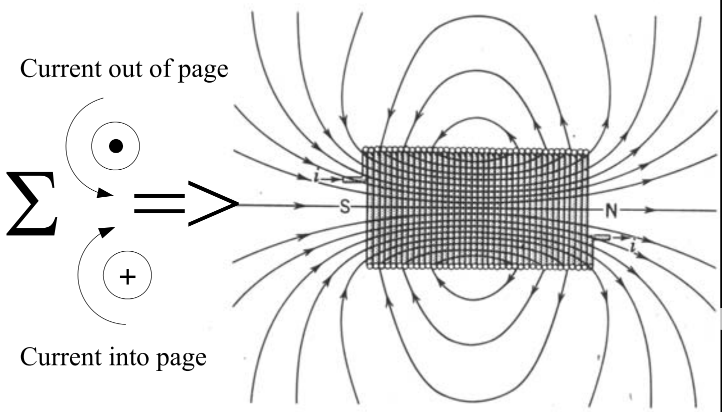
Is it worth your while to try and wind your own solenoid and use it in a plunger-type actuator? What is a good application for a solenoid in your machine? Do the characteristics of solenoids trigger any ideas?

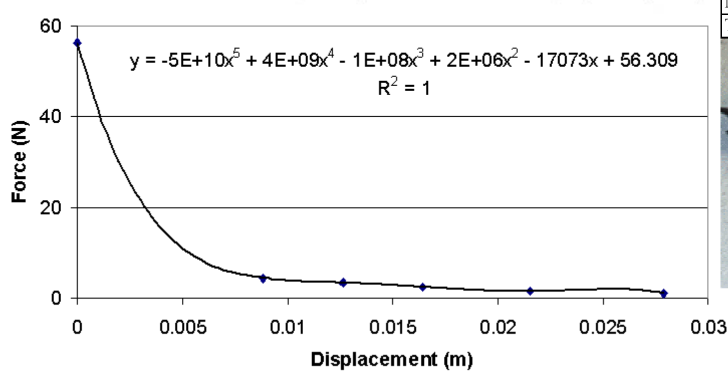
2. Fringing fields and limited heat transfer (coils get hot and melt!) make it difficult to accurately determine the force on a solenoid plunger from this simple calculation, but solenoid manufacturers provide good data for their products, and many standard sizes are available. For application examples and typical solenoids, see for example http://www.ledex.com/df/LXGOG/index.html

Magnetic Circuits: *Solenoids*

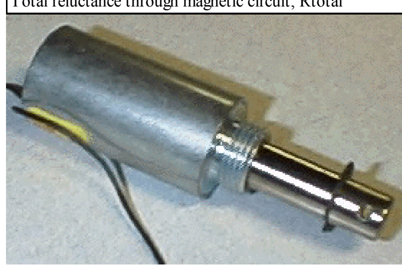
- Solenoids use a coil to generate a magnetic field that then attracts an iron plunger
  - They have limited pull force and stroke and are often used to open or close devices
    - Understanding their physics of operation helps to ensure that you will not improperly use them
    - Motion of the plunger into the case reduces the potential energy and hence creates a force
    - The force drops off rapidly (nonlinearly) with motion of the plunger
  - The force/displacement curves can be predicted, but typically just use curves from the manufacturer (e.g., www.ledex.com)

From D. Halliday & R. Resnick, *Physics Parts I & II Combined 3rd edition*

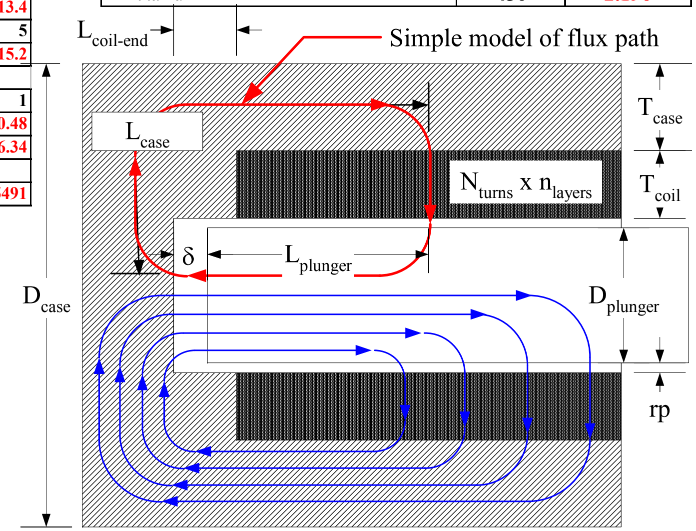




Solenoid_force.xls	
To estimate pulling force of a solenoid plunger	
By Alex Slocum & Jeff Lang	
Last modified 6/22/03 by Alex Slocum	
Enters numbers in **BOLD**, Results in **RED**	
Coil	
Current, i (amps)	**2**
Number of turns, Nturns	**200**
number of layers, nlayers	**1**
Magnetomotive force, Fmcoil (ampere-turns)	400
wire diameter, dwire (mm)	**0.25**
Length of coil, Lcoil (mm)	50
Total length of wire, Lwire (mm)	9708
Coil material (assumed copper, see below) resistance μΩmm	**17**
Wire resistance, Rwire (Ohms Ω)	3.36
Power dissipated as heat (Watts)	13.4
Distance coil end to case bottom (>delta max), Lcoil_end, (mm)	**5**
Inside diameter, Dcoil (mm)	15.2
Plunger	
Plunger axial displacement, delta (mm)	**1**
Magnetic flux density across plunger end gap, Bgap (tesla)	0.48
Resulting force F (N)	16.34
Magnetic circuit path elements:	
Total reluctance through magnetic circuit, Rtotal	4695491



Wire_resistivity.xls		
To estimate resistance of a length of solid wire		
By Alex Slocum & Jeff Lang		
Last modified 6/22/03 by Alex Slocum		
Enters numbers in **BOLD**, Results in **RED**		
wire diameter, D (mm)	**0.5**	
wire length, L (mm)	**1000**	
Resistivity:	μΩ-mm	R (Ohms)
gold	**24**	0.122
aluminum	**27**	0.138
copper	**17**	0.087
silver	**16**	0.081
tungsten	**56**	0.285
titanium	**430**	2.190



Lorentz Forces

Another means to generate forces is to use a charge in a magnetic field. The details of this simple effect were studied in great depth by many people; however, because Hendrick Antoon Lorentz (1853-1928) made so many significant contributions to the study of electric and magnetic fields, the equation describing the force on a charged particle is called the *Lorentz equation*:[1]

$$\mathbf{F} = q_o\mathbf{E} + q_o\mathbf{v}\times\mathbf{B}$$

The first part of the Lorentz equation is concerned with electrostatic attraction between charged plates. The second part can be applied to predict the forces **F** caused by a magnetic field of strength **B** on a wire of length **l** that is carrying a current i:

$$\mathbf{F} = i\mathbf{l}\times\mathbf{B} \qquad F_{\text{Force on a straight wire of length } l \text{ in a transverse B field}} = ilB$$

l is a vector whose magnitude is the length of the wire that points in the direction of the current flow. The right-hand-rule can be applied to find the force on a current carrying wire in a magnetic field: If the index finger points along the wire in the direction of current flow (from + to -), and the middle finger is aligned with the magnetic field (from North to South), then the magnetic force will be in the direction of the thumb.

Perhaps the most common use of Lorentz force actuators is as loudspeakers which must have high bandwidth and move large amounts of air. Note that there are two forms of such loudspeakers:

- *Voice-coils* move a coil of wire in a magnetic field, and they are described by the second term in the Lorentz equation. *Page 7-21 shows how a Lorentz force actuator is used to control small high speed motions of a CD drive's optical pickup unit.*
- *Electrostatic actuators* generate attraction forces between two charged surfaces in the direction of potential motion between the surfaces.

1. See http://www.sciencejoywagon.com/physicszone/lesson/otherpub/wfendt/lorentzforce.htm

The former are used for modest bandwidth loudspeakers/motion systems, and can move over many millimeters as would be required to create respectable power[2]. At higher frequencies, lower moving mass and amplitudes are required, which can be better realized with an electrostatic actuator. Such actuators are used in high-end speaker's tweeters. Electrostatic attraction is also used to hold semiconductor wafers to chucks during processing, and in *Micro Electro Mechanical Systems (MEMS)* to actuate components. Two common electrostatic actuators are the *comb drive* and the *capacitor plate*. Both operate on the principle that the energy U stored in a capacitor is a function the capacitance, with overlap x of the teeth (depth t) or separation h of plates (area L x t) respectively, and the voltage squared. Since the mechanical force is the derivative of the energy with respect to motion:

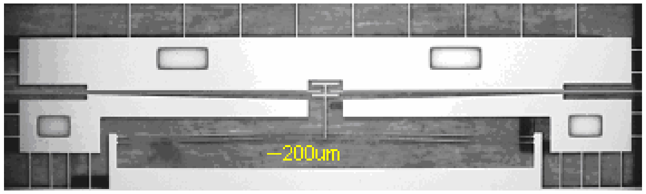
$$U_{\text{Comb Drive}} = \frac{1}{2}\left(\frac{\varepsilon_0 N t x}{h}\right)E^2 \qquad U_{\text{Capacitor Plate}} = \frac{1}{2}\left(\frac{\varepsilon_0 L t}{h}\right)E^2$$

$$F_{\text{Comb Drive}} = \frac{\partial U}{\partial x} = \frac{\varepsilon_0 N t E^2}{2h} \qquad F_{\text{Capacitor Plate}} = \frac{\partial U}{\partial h} = \frac{\varepsilon_0 L t E^2}{2h^2}$$

The force limiting factor is plate separation. In a MEMS system, where the dimensions are small even with respect to the forces often desired, the obtainable separation often makes electrostatic force insufficient. A *zipping actuator* overcomes this problem with the use of a thin compliant starting cantilever that is attracted at low voltage to close the gap. Shown are different stages of a novel MEMS electrostatic actuator with four different voltage excitations[3]: a) zero applied voltage, b) voltage U applied to the lower electrode (1) bends the thin starting cantilevers (2) up to close the gap (3), c) voltage U zips the actuator beam (4) to push on the bistable beam (5), d) voltage U applied to the upper electrode (6) returns the bistable beam to its initial position.

Can you use simple magnets and wind your own coil to create a Lorentz force actuator? Is it useful as an actuator for a trigger? Why is it not likely that you would make your own motor? Would an electrostatic actuator be useful for lifting an object such as a piece of paper?

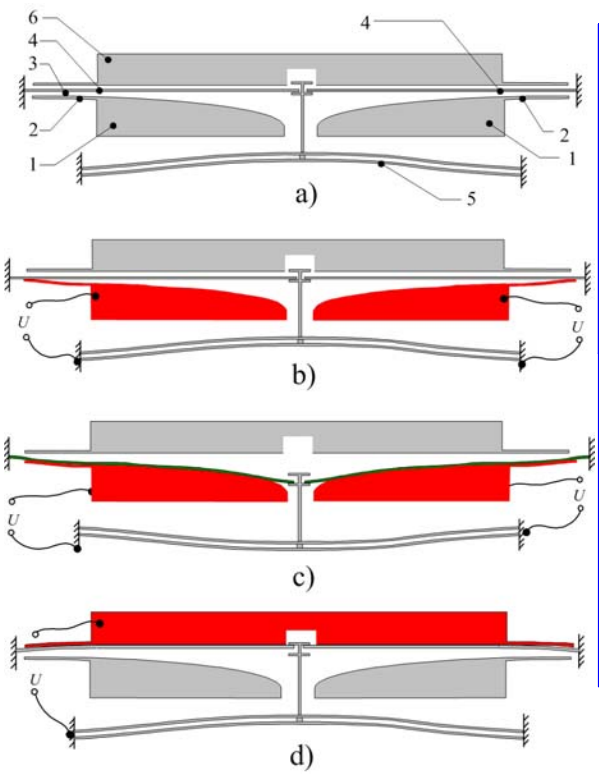
2. Large numbers of them can make your ears seem like they are on fire!
3. See for example 48)Li J., Brenner M.P., Lang J. H., Slocum A. H., Struempler R., "DRIE-Fabricated Curved-Electrode Zipping Actuators With Low Pull-in Voltage," in Proc. 12th International Conference on Solid-State Sensors and Actuators (Transducers '03), Boston, USA, June 8-12, 2003, pp. 480-483.



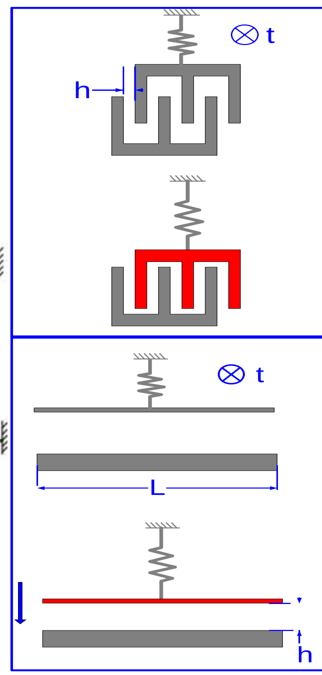
Hendrick Antoon Lorentz
(1853-1928)

Lorentz Forces

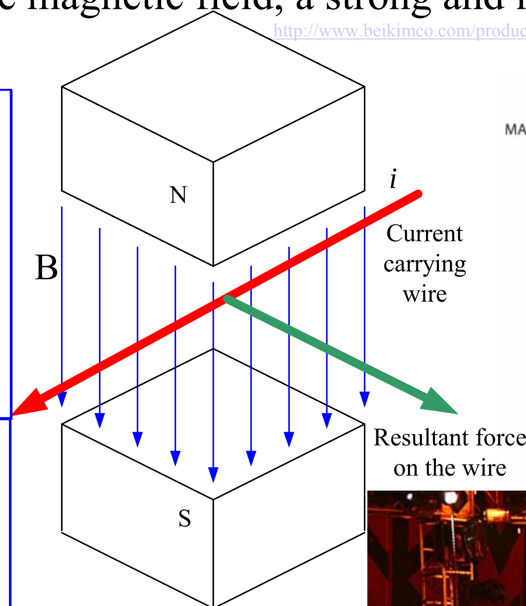
- Lorentz forces are created either from differential charges between objects (from Coulomb's law) or by a coil in a permanent magnet field
  - With a uniform magnetic field over a wide region, a reasonably constant force can be generated in a current carrying wire that moves within the field
    - As long as the wire stays totally in the magnetic field, a strong and linear bi-directional force is created

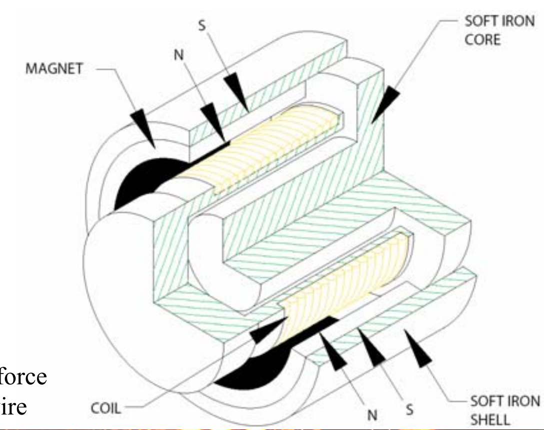


Dr. Jian Li's starting cantilever zipping actuator (from his Ph.D. thesis)

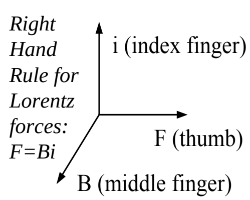


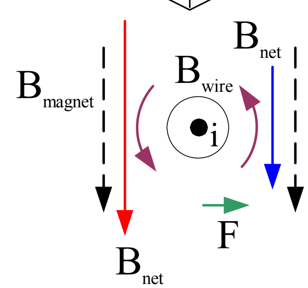
From BEI KImco's website: http://www.beikimco.com/products/actuator/guide/BEIKimco-VCA-AppGuide2002.pdf#Theory





The POWER of Lorentz force actuators unleashed!
(from Wango Tango - Rose Bowl, Pasadena, CA, May 17, 2003 Photo by Jay Gilbert http://www.kissonline.com/news/index.php?page=news_index.html)





Electric Motors

Electric motors freed industry from having to be located near rivers, and thus helped industry to spread across the land. There are many types of electric motors, but all operate on the principle that when a conductor carrying an electric current is in a magnetic field, a force is generated on the conductor (and by reciprocity on the source of the magnetic field). The magnetic field can be created permanent magnets or an electric field. Because there are so many ways in which this type of force can be created, and there are so many different applications, many different types of motors have been developed over the years.

Direct current (DC) machines were a natural first outcome of Michael Faraday's (1791-1867) discovery that a magnet that moved into a coil caused a potential difference at the terminals of the coil. The potential difference, or voltage, was proportional to the speed of the coil with respect to the magnetic field, and the current generated was proportional the strength of the field and the number of coils. This discovery enabled dynamos (electric generators) to be built. Early generators used reciprocating coils of wire and were not very efficient because when a coil is simply rotated in a magnetic field, the current flow reverses. At that time no one knew how to use this alternating current because AC motors were not yet invented, so "useful DC power" was only generated for half the cycle. Necessity is a parent of invention, and the principle of commutation was invented in 1859 by Pacinotti to switch the motor leads as the coils turned, and thereby provide efficient generation of DC current. In reverse, commutation caused the current direction to switch in a coil of wire in a magnetic field, which would cause the coil to rotate, and the DC motor was born.

Early commutators were not very efficient, until Zenobe Gramme (1826-1901) patented a ring armature machine with a new type of commutator that enabled it to achieve very high efficiency in 1870. The DC electric revolution was now on. Werner von Siemens (1816-1892) led the charge with numerous innovations and the formation of a company that bears his name to this day. Soon thereafter, George Westinghouse (1846-1914) used his success with the first air brake to expand into other areas including large DC electric motors.

However, DC power was limited in the distances it could be efficiently transmitted. Fortunately, Nikola Tesla (1856-1943) invented the polyphase system of generating alternating current (rotating magnetic fields), but he was rebuffed by Thomas Edison (1847-1931). Later while working for George Westinghouse (1846-1914), Tesla perfected his polyphase generation system and invented numerous other AC machines. Even though Edison started the electric revolution with his direct current (DC) generating systems, it was Tesla's brilliance that led to efficient AC power becoming the standard for power transmission. As AC power took over, Edison did not stand still, and by 1890 Edison had organized his various businesses into the Edison General Electric Company which later became General Electric[1]. Edison's counter to the brilliance of Tesla was perhaps the even greater brilliance of Charles Steinmetz (1865-1923) who invented numerous AC machines as well as developed the theory of complex numbers for the analysis of AC systems.

Different types of motors have different torque-speed responses, but, the most common type of motor used in small machines, particularly those created for design contests, are *DC brushed motors*. This is due to the fact that the speed of a DC motor can be easily varied simply by varying the input voltage. The magnetic field is created either with permanent magnets (most common for small motors) or windings that are incorporated into the fixed housing (the *stator*). Carbon brushes switch the current between windings on the *rotor* to keep the current flow in proper phase with the magnetic field as the rotor rotates.

The wire in a motor winding, which has a finite resistance R, can only carry so much current I before it melts (Power dissipated = I^2R), see *wire_resistivity.xls,* but it is generally not limited in how much voltage it can carry. Therefore, one can get more power out of a motor before one melts it, by running the motor at high voltage (speed) and low current (torque). A transmission can then be used to reduce motor speed and increase its torque. Remember, the goal is to minimize cost of the overall system!

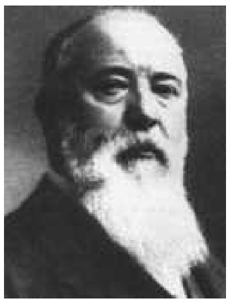
Take apart an old appliance or tool, such as a blender or a drill. What kind of motor does it use, and how does it attain various speeds?

1. http://www.ge.com/en/company/companyinfo/at_a_glance/history_story.htm

- Electric Motors
  - DC (direct current)
    - External field
    - Permanent Magnet
      - Brushed (commutated)
      - Brushless
  - AC (Alternating current)
  - Stepping Motors

Electric Motors

Werner von Siemens
(1816-1892)



Zenobe Gramme
(1826-1901)

- **Electric motors convert *voltage* into *speed* and *current* into *torque:***

  $\omega = K_b * V \qquad \Gamma = K_T * I \qquad K_b = K_T$ (mks units)

  – Different types of motors have different torque-speed responses

  – Remember, *power* is the product of *torque* Γ and *speed* ω: Watts=N-m*rad/sec

- **High current creates heat in the motor:**

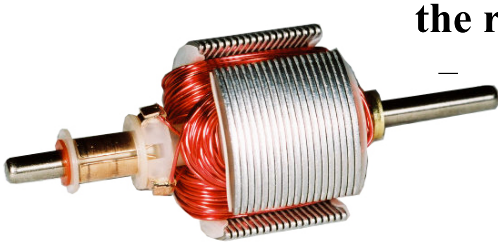
  – Power dissipated in the motor windings is equal to the product of the current squared and the winding electrical resistance R: $P_{thermal} = I^2 R$

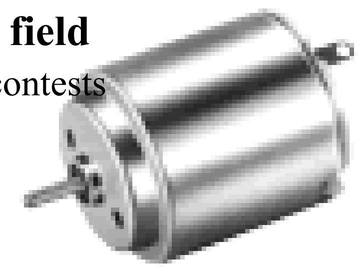
  – Total electrical power = current X voltage, because some power is also dissipated in the magnetic circuit

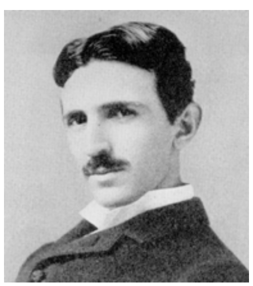
  – It is often more efficient to get high torque/low speed from a high speed motor coupled to a gearbox

- ***DC brushed motors* often use carbon brushes to provide current to coils on the rotor to create electric fields perpendicular to the magnetic field**

  – Linear torque-speed response and are commonly used in robot design contests

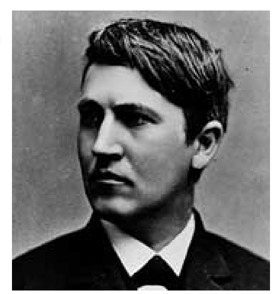




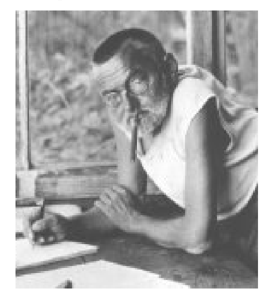


Nikola Tesla
(1856-1943)

George Westinghouse
(1846-1914)



Thomas Edison
(1847-1931)



Charles Steinmetz
(1865-1923)

Electric Motors: DC *Brushed Motors*

One of the lowest cost motor systems to use is a permanent magnet DC brushed motor, where the stationary permanent magnets create a magnetic field through which pass current carrying coils on the rotor. The ends of the *coils* are connected to bars that make contact with the stationary brushes, often made from carbon or copper. Voltage (or current) applied to the brushes flows through the coils[1]. As shown, given a nominally uniform magnetic field through the cross section of the motor, the current goes into the coil on one side of the motor center, through end turns, and then out of the coil on the other side of the motor center. By the right-hand-rule for current (index), magnetic field (middle) force (thumb), the current in the coil creates a torque on the rotor.

Shown is a 3-pole motor: there are three conductors to which the stationary brushes[2] provide current depending on the angle of the rotor. As shown on the left, there is a position where the brushes make contact with only two of the commutator conductors, but due to the manner in which the windings are attached to the commutator's conductors, all the windings are energized. In the figure, a "+" indicates current flow into the page, and a "•" indicates current flow out of the page. Since the magnetic field is nominally from the top (N) to the bottom (S) across the cross-section of the motor, current flowing into (+) the page produces a force to the left. Current flowing out of the page produces a force to the right. Whether the force adds to or subtracts from the intended torque direction depends on if it is located above or below the equator line of the motor (shown by the dashed black line). Here, the red arrows indicate induced forces that subtract from the desired torque. Green arrows add to the desired torque. The further the forces from the equator line, the greater their contribution to the net torque.

As the rotor turns, the commutator brushes make contact with two of the commutator conductors. As shown, this results in one set of windings not being energized for a moment, but there is still significant torque generated.

1. Brushless DC motors (or electrically commutated motors or ECMs) have their magnets on the rotor, and their windings on the stator. The motor driver senses the rotor magnet position and then switches current to the coils. This is a more expensive system, but it can run at stall without danger of brushes arcing and burning. Brushless DC motors are often used for position control servo systems and where sparks might cause a fire. They are rare in basic robot design contests, and will not be considered further herein.
2. For illustrative purposes, the brushes are shown (blue and green) on the inside of the commutator circle, but as the photos show, they are actually located on the outside).

However, there will be some torque difference, known as *torque ripple*, and thus the goal of a motor designer is to minimize torque ripple. This can be done by adding more poles to the motor and slanting the windings. The end turns of the windings do not contribute to the generated torque; hence by reducing torque ripple by adding windings and commutator segments (poles) one decreases efficiency and increases cost. Designing an efficient motor is clearly a significant challenge.

When creating a power budget for the system to evaluate battery requirements, compare the stall power and the normal operating power:

$$P_{\text{motor electrical power}} = \frac{\Gamma_{torque}\omega_{speed}}{\eta_{\text{motor efficiency}}} \approx I^2_{\text{current}} R_{\text{motor resistance}} + \Gamma_{torque}\omega_{speed}$$

$$P_{\max} = I^2_{\text{stall}} R_{\text{motor resistance}}$$

The maximum electrical power occurs at zero speed, and mechanical power is the product of torque Γ and speed ω (Watts = N-m*rad/sec). Thus when maximum electrical power is being consumed by the motor, no mechanical power is being generated. Conversely, when the motor is spinning at its maximum speed, the no-load speed, no mechanical power is being generated because the torque is zero. In fact, at the maximum speed, the spinning windings in the permanent magnetic field are generating a voltage opposite to the voltage being supplied to the windings. This *back EMF* is what limits the speed of the motor. However, if too high an input voltage is supplied to the windings, without current limit, the magnetic field that is also created by the windings acts in an opposite direction to the permanent magnets' field, and it can cause them to become demagnetized.

As shown, DC permanent magnet motors have a relatively linear torque-speed curve when the voltage is varied in order to change the speed. As also shown, this means that their power profile is nominally parabolic. However, the true parabolic profile is skewed by electrical losses, and the obtainable power is equal to the product of the torque, speed, and efficiency of the motor. In the next section, motor design parameters will be discussed.

How would you minimize the electrical power while maximizing the mechanical power? How does the transmission ratio fit into this assessment?

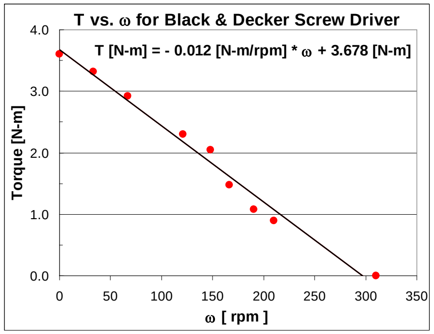
Electric Motors: DC *Brushed Motors*

- For a DC motor with a linear torque-velocity relationship, maximum power is obtained when the motor runs at ½ maximum speed:

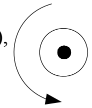
$$\Gamma = S\omega + \Gamma_{max} \qquad S = {-\Gamma_{max}}\big/{\omega_{max}}$$

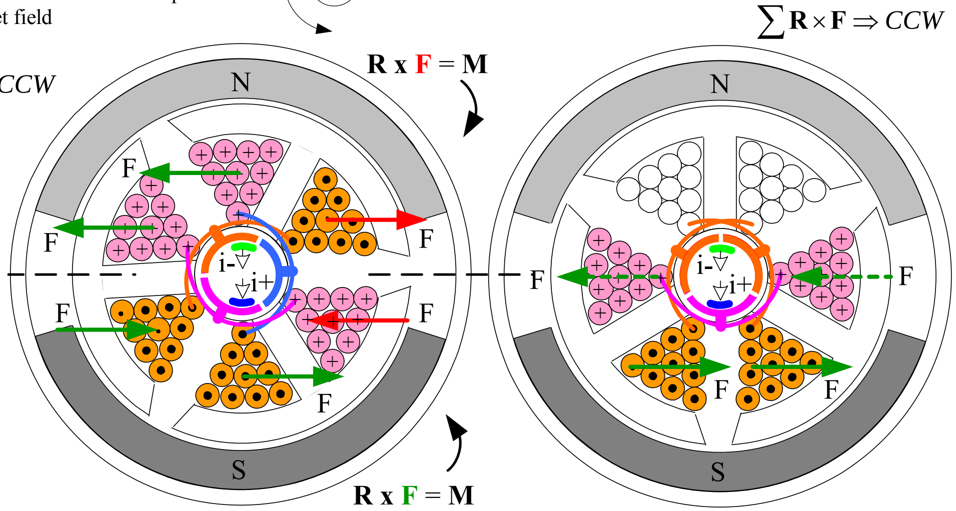
$$P = \Gamma\omega = S\omega^2 + \Gamma_{max}\omega$$

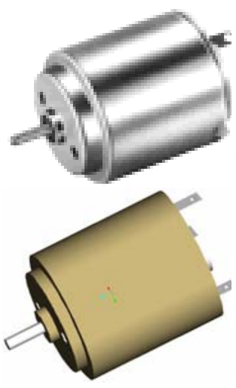
$$\frac{\partial P}{\partial \omega} = 0 = 2S\omega + \Gamma_{max} \quad \Rightarrow \omega = \frac{\omega_{max}}{2}$$



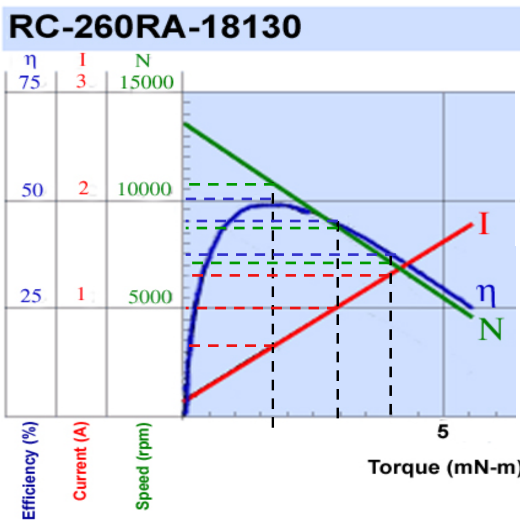
Current out of the page creates a CCW magnetic field (right hand rule), which distorts the motor's permanant magnet field

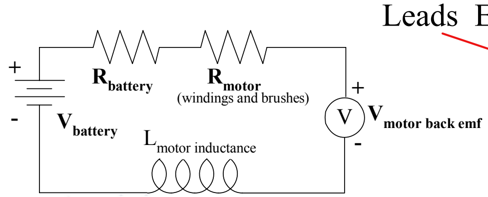


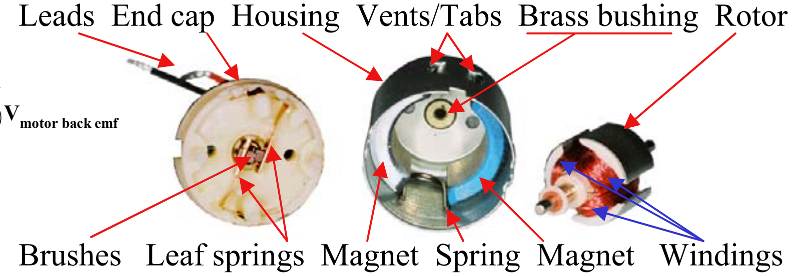




Motor solid model







MODEL	VOLTAGE		NO LOAD		AT MAXIMUM EFFICIENCY					STALL		
	OPERATING RANGE	NOMINAL	SPEED	CURRENT	SPEED	CURRENT	TORQUE		OUTPUT	TORQUE		CURRENT
			r/min	A	r/min	A	mN·m	g·cm	W	mN·m	g·cm	A
RC-260RA-18130	4.5 ~ 6.0	4.5V CONSTANT	9800	0.14	7750	0.53	1.48	15.1	1.20	7.06	72	2.00
		6V CONSTANT	13600	0.17	10840	0.67	1.71	17.5	1.94	8.44	86	2.62

Resistance across motor leads 3-5 Ω new, 1-3 Ω after wear-in!

DC Brushed Motors: *Gearmotors*

It has been shown that it can be a very good thing if a motor's output is directed into a transmission (i.e., a gearbox) with the "optimal" transmission ratio. There are many different types of gearboxes that can be used in conjunction with a motor, and when they are sold together, they are often referred to as a *gearmotor.* Perhaps the most common transmission for use with small motors is the planetary gear transmission which was discussed in detail on page 6-16. Modular planetary gear transmissions are popular because they can be assembled in stages, and many different transmission ratios can be obtained from a kit. A typical assembly procedure for a modular planetary transmission, such as the one shown, includes:

- Make sure there is no "flash" on any of the parts being assembled. (Flash is the stray plastic left over when removing the plastic parts from the trees, or from overmolding).
- File down the hub on the last planetary output stage so the output shaft drive tangs will not spin on the surface at high torques. Make sure to debur and clean off all the particles. However, there is now no maximum torque protection, so too much external torque could strip the gear teeth. Either consider an external adjustable slip clutch, or just be careful!
- In a clean area with clean hands, with all the parts clean and ready to assemble, begin the assembly process.
- Put a small dab of lubricant on each planet carrier post prior to sliding on the planet gears. Notice that the carriers rub on an inside flange of the grey housing. This is a point of high resistance torque. So you should also grease these surfaces.
- Put a small dab of lubricant on each gears' teeth, and each gear's face. Remember, too much lubricant attracts dirt and causes viscous shear losses.
- Assemble the stages, and then assemble the stages into the housings, taking care to note that the housing exterior sections have alignment marks.
- Test each stage as it is assembled: Ideally you will be able to back-drive the gearbox by turning the output shaft.
- For the final stage, place two washers between the output shaft's tanged metal plate and the inside of the housing to ensure that the drive tangs stay engaged with the stage's grooves.
- When you tighten the screws that hold the housings together, do not CRUSH the housings. This will bind the gears and destroy the gearbox. Snug up the nuts and then apply hot-melt-glue to the nut/bolt interface to ensure they do not vibrate loose.
- If you are face mounting your motors (screwing them to a plate using the three housing assembly screws) DO NOT bend the housing screw "ears". This can distort the housings and bind the gears. Put washers under them before you screw the motors down.
- Four stages are provided (4:1, 4:1, 5:1, and 5:1) which means that you can achieve 4:1, 5:1, 16:1, 25:1, 80:1, 100:1, or 400:1 ratios. However, you might be able to swap stages with other people (or your other gearmotors if you only need two units) and get ratios of 64:1 or 125;1....be creative!

If all of above steps are done carefully, high torque and good efficiency can be obtained from the modular gearmotors.

A design engineer must often make a quick feasibility calculation regarding the size of a motor/gearbox. For example, given some typical proportions between the motor housing length and diameter, their relation to the rotor length and diameter, and an estimate of the effective "electromagnetic shear strength" in the rotor/stator annulus that can be achieved:

$$\Gamma_{\text{estimated}} = \tau_{\text{electromagnetic shear stress}} \times \frac{D_{\text{motor housing}}}{4} \times \left(\pi \times \frac{D_{\text{motor housing}}}{2} \times \frac{L_{\text{motor housing}}}{2} \right)$$

$$\Gamma_{\text{estimated (N-mm)}} \cong \frac{D^2_{\text{motor housing (mm)}} L_{\text{motor housing (mm)}}}{500_{\text{for } \tau_{\text{electromagnetic shear stress}} = 0.1 \text{ bar for low cost motors}}}$$

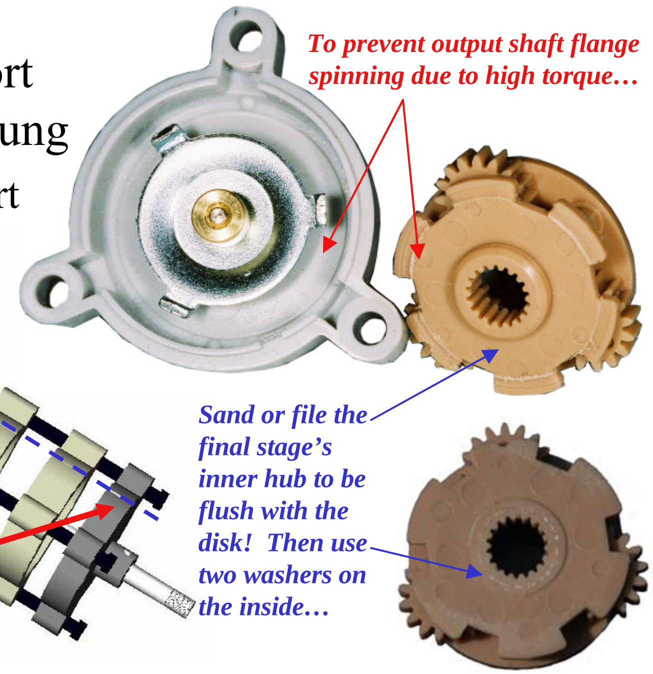
The spreadsheet *Gearmotor_size_estimator.xls* is a more general form of this estimation, and includes terms for the gearbox. Other types of gearmotors include *servos*[1] used in radio controlled cars. The servos have internal sensors and closed-loop analog servo loops so their output shafts will move to the desired input position or velocity. High torques and good angular position capability can be obtained.

Your gearmotors should be ready to test on a simple vehicle. Run some experiments and see if the time to move across the contest table is about what the spreadsheet predicts.

1. See for example www.towerhobbies.com

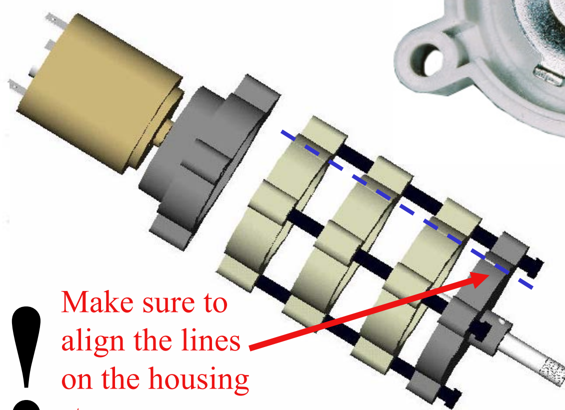
DC Brushed Motors: *Gearmotors*

- Integrating the transmission with the motor saves space and the need for a coupling
  - Wormgear drives are used to power electric automobile seats and windshield wipers (see page 6-23)
    - High transmission ratio, single stage, not back-driveable
  - Planetary transmissions are often back-driveable (see page 6-15)
- Output shafts and their support bearings will generally support small robot machine radial loads if they are not too far overhung
  - In general, be extremely careful when using output shaft support bearings to support radial loads!

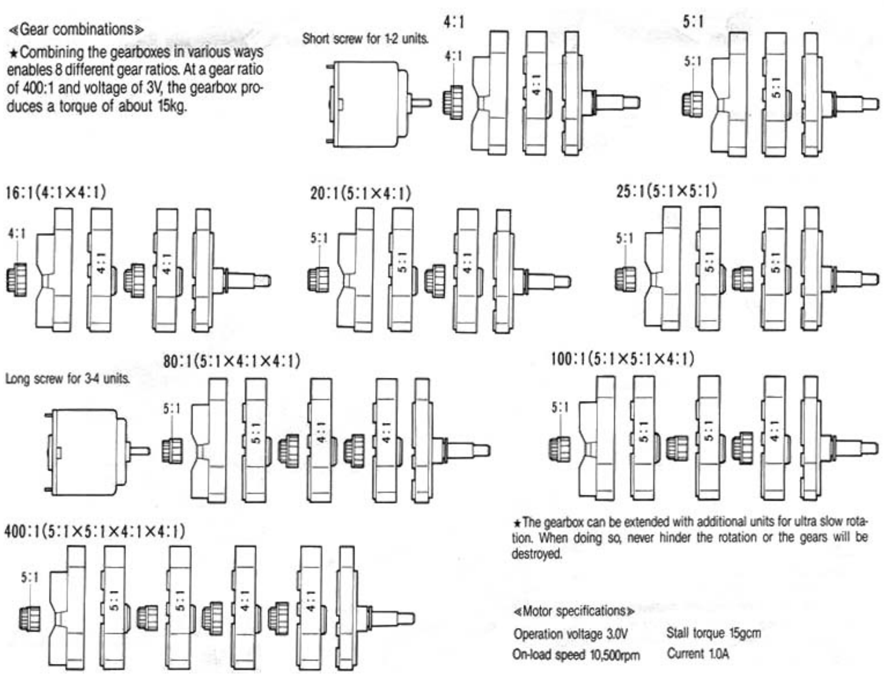


To prevent output shaft flange spinning due to high torque…

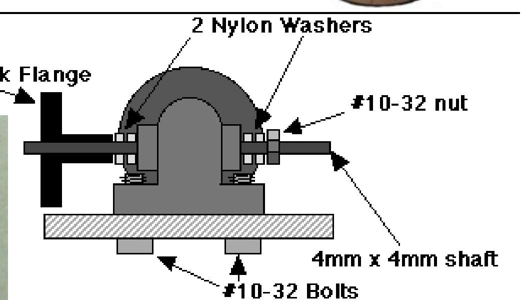
Sand or file the final stage's inner hub to be flush with the disk! Then use two washers on the inside…



! Make sure to align the lines on the housing stages



Bosch automotive power seat control motor-wormgear



DC Brushed Motors: *Best Operating Region*

The useful operating region of a motor is the region where the motor is operated with good electrical and mechanical efficiency. This means the motor should not be run too fast or too slow, or the engineer is not getting good value for the cost of the motor.

The *maximum voltage* (and hence current) at which a motor operates must not induce a magnetic field in the windings which too greatly opposes, and hence demagnetizes the permanent magnets. (The motor rotor should also not be taken out from a motor or demagnetization may also occur). The maximum voltage also restricts the maximum speed, which is further limited by how fast the brushes can run on the rotor before aerodynamic lift makes them float. As shown, this value typically falls in a range near the speed of maximum efficiency, but should still be checked. Likewise, the *minimum voltage* should not be less than 1/2 the maximum voltage, or the motor selected is too large for the application and too much money is being spent.

The *maximum mechanical power* that can be obtained from the motor was shown to occur at one-half the maximum speed or torque. Rarely can one operate at exactly this value, but since the curve is parabolic, one can operate in the region and still achieve a good design.

The *thermal duty cycle* is a very important parameter, because the primary cause of motor failure is too much torque for too long a time. This means the motor draws a lot of current i, and thermal power dissipated in the windings and brushes with resistance R equals i^2R. *In addition, due to the fact that the winding resistance increases with temperature, the motor torque will fall by 0.4%/^{o}K temperature increase from ambient!* Most motor manufacturers provide maximum torque values for their motors based on different duty cycles. The maximum torque for a 50% duty cycle, T_{50}, is typically given, and T_{100} = $T_{50}/1.414T_{50}$, and $T_{25} = 1.414T_{50}$

The overall efficiency, η_{motor}, is a function of how much mechanical power $P_{motor\ mech}$ is generated verses how much power, P_{diss}, is dissipated by electrical, magnetic, and mechanical losses. *Electrical losses* include the power dissipated from the windings and brushes (i^2R), the brushes' contact voltage drop u_{brush} ($u_{brush}i$), and the commutation losses from the brushes breaking contact with a winding (inductance L) and dissipating the magnetic field energy stored ($\frac{1}{2}Li^2$) before they make contact with the next winding. *Magnetic losses* include eddy current and hysteresis losses in the iron core rotor. *Mechanical losses* include brush and bearing friction, and air resistance. All these factors are typically determined by the motor manufacturer[1].

Most motors are operated at 25%-50% of the maximum stall torque.

The efficiency of the transmission system must also be considered. If your machine is to push with 10 N of force at 0.2 m/s, then it needs to deliver 2 Watts of mechanical power. If the transmission efficiency is η_{trans} = 50%, the motor mechanical power $P_{motor\ mech}$ needs to be 4 Watts. However, the motor efficiency is likely to be η_{motor} = 40%. Thus the control system must source 10 Watts of electrical power! *The control system typically must provide 3-5 times the required mechanical output power from the motor/transmission!* Thus for quick estimates of battery life, use a factor of 5 (overall system efficiency ν_{system} = 20%). The system design trade-off should now be clear: One can avoid transmission losses by using a motor to directly drive a load, however, this typically results in high torque and low speed, and very low motor efficiency. The goal for the system is to maximize overall system efficiency:

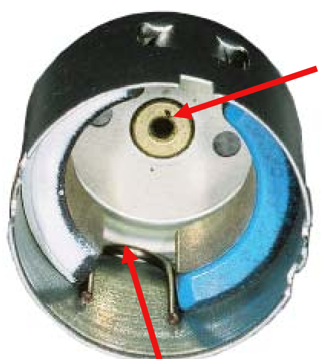
$$P_{\text{control system}} = \frac{P_{\text{required output}}}{\eta_{\text{transmission system}}\eta_{\text{motor}}} \qquad \eta_{\text{system}} = \left(\eta_{\text{transmission system}}\eta_{\text{motor}}\right)_{\text{maximized}}$$

If your motor does not come with efficiency curves, or you are unsure of transmission system efficiency, you can always select the optimal transmission ratio based on motor and load inertias, and then perform a bench level experiment where you measure the voltage and current going into the motor and the force and velocity applied to the load. This will provide you with the system efficiency. Set up and run such an experiment for a typical motor/transmission system in your machine. It is important to determine the system efficiency, so you can determine how much power the control system must deliver, and consequently this enables you to determine battery life.

1. Note that some motor manufacturers provide design curves with torque on the horizontal axes. No worries, just be glad when the motor manufacturer actually provides the efficiency data so you can use it to develop a good estimate of the power your control system must provide.

DC Brushed Motors: *Best Operating Region*

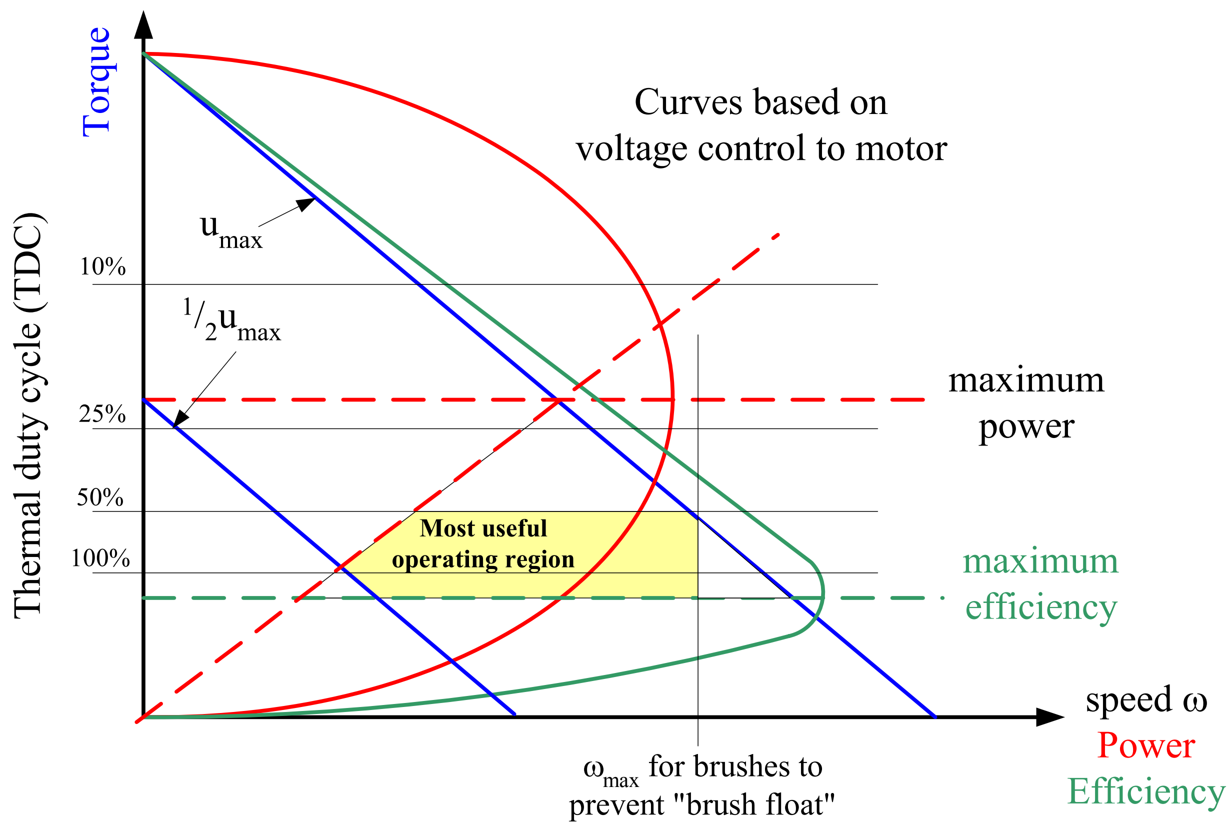
- When selecting a motor and transmission, the motor must not be too big nor too small:
  - The maximum operating voltage u_{max} (and hence maximum current) is set to prevent motor magnet demagnetization
    - To continually use a motor at less than ½ u_{max} is not cost effective
- ***HEAT*** (thermal overload) is one of the greatest causes of motor damage
  - Time must be allowed between on-cycles for the motor to cool down



Notice the bronze bushing bearing for front shaft support

Notice the "U-spring" that keeps the magnets in place inside the housing!

Motor mass: 26 grams
Rotor mass: 10 grams
Rotor inertia: 560 g-mm^2



DC Brushed Motors: *Design Spreadsheets*

DC brushed motors are commonly used in voltage control mode, where the voltage is varied and the controller provides as much current as is needed. They have a linear torque-speed response. The *Steepness* is a measure of the "goodness" of the motor, *s* = max torque/ max speed, and the steeper the better. With mks units, the motor constant *K* equals the torque constant K_T (N-m-amp^{-1}) and the back-EMF constant K_e (radian-Volt^{-1}-second^{-1}):

$$s = \frac{\Gamma_{\text{max (N-m)}}}{\omega_{\text{max (radians/sec)}}} \qquad K = K_t = \frac{\Gamma_{\text{max (N-m)}}}{i_{\text{max (amp)}}} = K_e = \frac{\omega_{\text{max (radians/sec)}}}{u_{\text{max (volts)}}}$$

The steepness is independent of the voltage *u* applied to the motor. The motor torque-speed curves are a series of shifted lines, each corresponding to the applied voltage. If the motor terminals are shorted together, the motor acts as a viscous brake, where the brake torque is proportional to angular velocity (radians/second). The mechanical time constant, a measure of how fast the motor can respond to an command to execute a step change in motion, is also a function of the steepness:

$$\Gamma_{\text{brake torque}} = s\,\omega_{\text{rotor speed (radians/sec)}} \qquad \tau_{\text{mech (seconds)}} = \frac{J_{\text{rotor inertia (kg-m\^{}2)}}}{S_{N-m-s/radian}}$$

To select the "optimal transmission ratio", the design engineer can start with the output power required, assume transmission and motor efficiencies will each be 50%, and select a motor whose product of maximum input current and voltage is 4X the system output power. From this family of motors, the designer gets a range of motor rotor sizes, and hence inertias from which to calculate a range of "optimal transmission ratios. Since the transmission is not yet selected, a reasonable estimate is to assume that the transmission system moment of inertia will be on the order of the motor rotor inertia, and the transmission system inertia is added to the motor inertia: in other words, double the motor rotor inertia when first estimating the optimal transmission ratio.

Once the optimal transmission ratio, and hence motor torque and speed, are determined, the design engineer can select the best motor to use. Given a family of motors, there will be different rotors and windings for a given limited range of housing sizes that can achieve different torque-speed curves *for the desired thermal duty cycle*. The motor should have a total power rating of *at least twice* the load power requirement in order to move itself and the load; however, remember, the motor and transmission efficiencies also need to be considered. Hence the required motor electrical input power is:

$$P_{\text{motor electrical input power } i_{\max}\bullet u_{\max}} = 2\frac{P_{\text{load}_{\max}}}{\eta_{\text{tranmission}}\,\eta_{\text{motor}}}$$

The *motor power rate* is a measure of the electrical to mechanical power conversion efficiency of a given actuator, and the units are *Watt/second.* The power rate combines both the motor's mechanical power output and mechanical time constant into a single figure-of-merit:

$$PR_{\text{motor}} = \frac{\Gamma^2_{\text{motor maximum torque at desired duty cycle}}}{J_{\text{motor inertia}}}$$

When the motor and load inertias are matched, the required motor power rate should be at *least* four times the load power rate. Conversely, one can determine the load power rate, and then select a motor with at least four times greater power rate. This yields the motor inertia from the catalog and the optimal transmission ratio can then be checked against the earlier estimate:

$$PR_{motor} = \frac{\Gamma^2_{motor}}{J_{motor}} \geq \frac{4PR_{load}}{\eta_{\text{transmission efficiency}}}$$

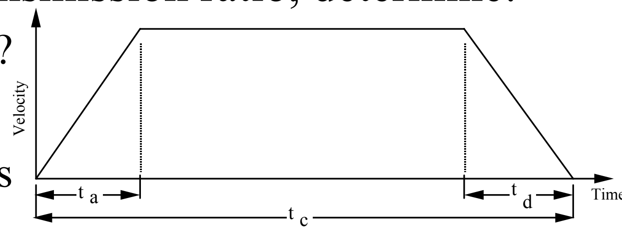
$$PR_{\text{load}} = \left(J_{\text{load}}\alpha_{\text{max acceleration}} + \sum \Gamma_{\text{friction \& external loads}}\right)\alpha_{\text{max acceleration}}$$

$$PR_{\text{load}} = \left(M_{\text{load}}a_{\text{max acceleration}} + \sum F_{\text{friction \& external loads}}\right)a_{\text{max acceleration}}$$

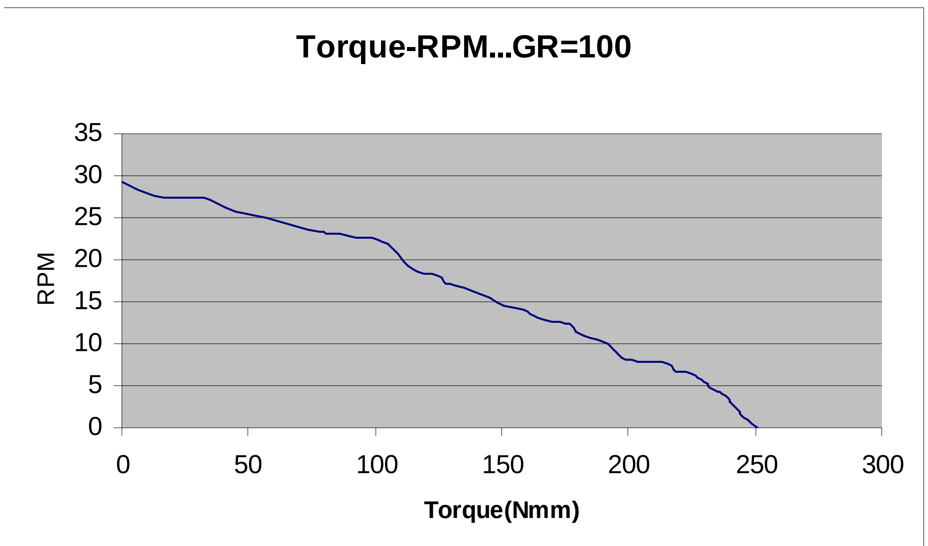
These methods provide the design engineer with means to initially select and check motor size. Use these methods to determine of your motor will allow your design to achieve the desired goal! Experiment with the spreadsheet *Gearmotor_move.xls* and also compare its more detailed output to the simple first order estimates obtained with *Power_to_move.xls*

DC Brushed Motors:
Design Spreadsheets

- Use the *Matched Inertia Doctrine* to find the "optimal" transmission ratio
- Motor power rate should be > load power rate
- With an obtainable transmission ratio, determine:
  – Will the wheels slip?
  – Move times
  – Battery requirements



- Play "what-if" scenarios with the spreadsheet *Gearmotor_move.xls*



Gearmotor_move.xls		
To estimate inertia of gearmotor and find system optimal transmission ratio		
By Alex Slocum		
Last modified 8/22/03 by Alex Slocum		
Enters numbers in **BOLD,** Results in **RED**		
Motor (torque and speed are NOT at absolute max values, but rather at max efficiency)		
Rotor mass, Mr (grams, kg)	**10**	0.0100
Diameter, Dm (mm, m)	**15**	0.0150
Mength, Lm (inches, m)	**12**	0.0120
Number of drive motors, Nm	**2**	
Nm Motors' rotary inertia, Jmotor (kg-m^2, g-mm^2)	5.63E-07	563
Motor operating efficiency, etamotor	50%	
Max motor torque, gammax (m-N-m, N-m)	**8**	0.008
Max motor speed, wmax (rpm, rad/s)	**13500**	1414
Motor speed at maximum efficiency, wmaxeff (rpm, rad/s)	**11500**	1204
Steepness S (N-m-s/rad)	5.659E-06	
Planetary Transmission		
Planet carrier assembly mass Mplanet, (grams, kg)	**2.1**	0.0021
Planet carrier outer diameter, Dpod (mm, m)	**20**	0.0200
Planet carrier inner diameter, Dpid (mm, m)	**10**	0.0100
Number of stages, Nstage	**3**	
Efficiency per stage, etastage	**90%**	
Planetary total rotary inertia, Jplanets (kg-m^2)	3.94E-07	
Output shaft mass, Mouts (grams, kg)	**4**	0.0040
Output shaft diameter, Douts	**4**	0.0040
Output shaft rotary inertia, Jouts (kg-m^2)	8E-09	
Total Nm planetary transmissions' rotary uinertia, Jtrans (kg-m^2)	8.04E-07	
Transmisssion efficiency (includes car wheels), etatrans	66%	
Car		
Mass of car, Mcar (kg)	**4**	
Diamter of wheel, Dwheel (mm, m)	**75**	0.0750
Max car acceleration, acar (m/s^2, g)	0.98	0.10
External loads, friction... Fext (N)	**0**	
2Wd or 4WD, Nwd	**2**	
Coefficient of friction wheel-to-ground, mu	**0.2**	
Optimal Transmission ratio by Matched Inertia Doctrine		
Optimal transmission ratio, ntrans	64	
Confirm: Number of stages = # required to achieve desired ntrans	yes	
Actual transmission ratio to be used, ntranactual	**64**	
Actual equivelent linear inertia of motor and tranny, mtrans (kg)	4.0	
Total actual sysetm equivelent inertia, Mtotal (kg)	8.0	
Power rates		
Motors' total power rate, PRmotor	187.41	
Load power rate, PRload	3.86	
System goodness (should be >1): PRmotor/(4PRload/etatrans)	7.96	
Motion Results		
Start-to-stop travel distance (must be>Xaccel), Xdes (m)	**1.5**	
Max. potential tractive effort (even mass distribution), Ftraction (N)	3.92	
Max. motor tractive effort (even mass distribution), Ftractive (N)	17.92	
Can wheels slip?	yes	
Maximum theoretical car speed vmaxpot (m/s)	0.83	
Car speed at max motor η, vmaxeff (m/s)	0.71	
"Steepness", slinear (N-s/m)	-4.73	
Time to accelerate to speed at max motor η, taccel (seconds)	3.22	
Taylor series approx. time to accelerate to speed at max motor η	3.89	
Distance travelled during acceleration to max speed, Xaccel (m)	1.48	
Time to travel 1/2 desired travel distance, tohtd (seconds)	1.75	
Total travel time, ttotal (seconds)	3.49	
Approx. travel time (no velocity limit, Taylor series approximation)	3.49	
Battery Requirements		
Estimated maximum power draw from batteries, Pbat (W)	8.43	
Estimated maximum energy draw from batteries, Ebat (J=N-m)	29.5	

DC Brushed Motors: *Design Spreadsheet Inputs*[1]

The spreadsheet *Gearmotor_move.xls* is only as good as the data put into it. In such a spreadsheet, or any program, it is important to identify the primary components. Typically, rotary inertias are primary elements because the moment of inertia goes like the product of mass and radius squared. Wheels are thus large system inertias, but manufacturers typically do not provide inertia data. What if you are on a plane or bus and do not remember what is the formula for the moment of inertia of a wheel? All one has to remember is the basic fundamental form of the moment of inertia integral, and then derive the rest:!

$$dm = \rho w r d\theta dr$$

$$I = \int r^2 dm = \rho w \int_{R_1}^{R_2} \int_0^{2\pi} r^3 d\theta dr = \frac{\rho w \pi \left(R_2^4 - R_1^4\right)}{2} \Rightarrow \frac{\rho w \pi \left(D_o^4 - D_i^4\right)}{32}$$

Then, as usual, a spreadsheet is a useful tool for crunching numbers:

Wheel_inertia.xls			
To determine mass properties of a wheel			
By Alex Slocum, last modified 2/3/2005 by Alex Slocum			
Enters numbers in **BOLD,** Results in **RED**			
Density, rho (grams/mm^3)	**0.001**		
	rim	flange	hub
Outer diameter, Do (mm)	**100**	**80**	**20**
Inner diameter, Di (mm)	**80**	**20**	**10**
width, w (mm)	**24**	**14**	**36**
Volume	**67,858**	**65,973**	**8,482**
Total, V (mm^3)			**142,314**
Mass	**68**	**66**	**8**
Total, M (gram)			**142**
Inertia	**139,110**	**56,077**	**530**
Total, I (gram-mm^2)			**195,187**

1. Many thanks to my graduate students and friends Jian Li and Fadi Abu-Ibrahim for their help in developing the MATLAB code and generating the gearmotor torque-speed data.

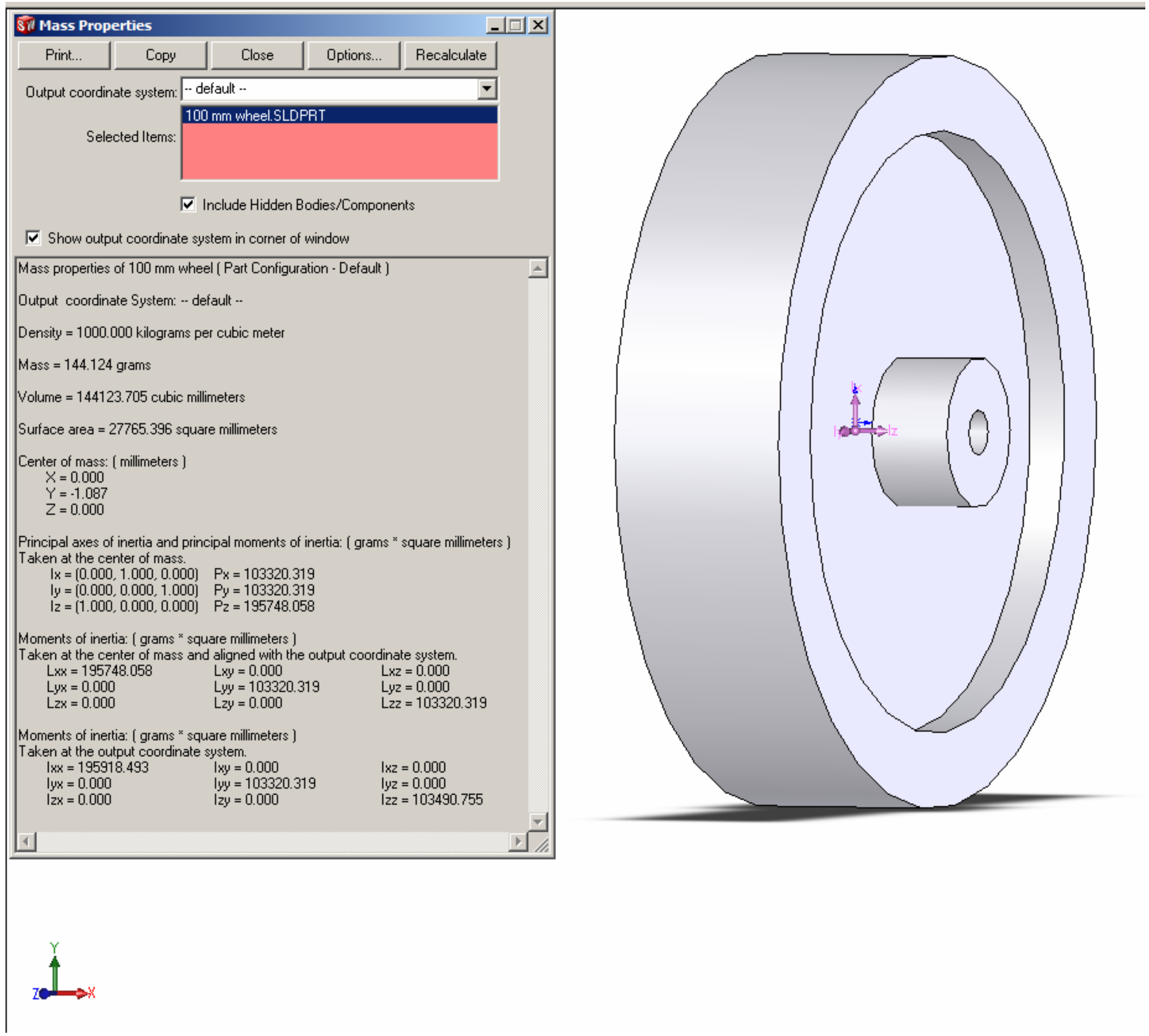
When the design is “done” and it’s time to create the solid model of the system, the mass property numbers obtained with the spreadsheet correlate with those from the solid model. This is a good thing. With the spreadsheet, one can far more rapidly play “what if” scenarios with regard to minimizing or maximizing component properties.

Philosophizing a bit, some folks say that engineers no longer have to know details or derivations because solid models are all-powerful; however, creative thought is often best done when walking in the woods....soaking in the tub... and knowing and understanding fundamental relationships and principles allows a design engineer to move faster and explore further than someone who is tied to a computer. Develop and use ALL the tools at your disposal including your brain as well as your computer!

Review your machine and try to rank order the masses and inertias with respect to their effect on system acceleration. Then use simple formulae and a spreadsheet to check your intuition. How good is your intuition? Then create the solid model of your system that will allow you to get an even more accurate estimate of the system mass and inertia properties. How good were your simple calculations? This sort of self-checking exercise is a fantastic way to develop your designer’s instinct!

DC Brushed Motors: *Design Spreadsheet Inputs*

- It is important to enter the proper mass property numbers into the design spreadsheets
- Solid models are very useful for getting good estimates of the mass properties
- They do not always exist at the early design stage
- Not all manufacturers have good mass property data
  - Go back to freshman physics to get the formulas and write a program to determine the inertia



DC Brushed Motors: *Motion Simulations*[1]

The spreadsheet *Gearmotor_move.xls* is a tool for a designer to get a quick look at the potential performance of the system. The equations used in the spreadsheet can also be used in a program, such as MATLAB™, where conditional non-linear effects are added which require numerical integration. Starting with the torque speed relation for the gearmotor and using the wheel radius $D_{wheel}/2$, determine the "steepness" s of the force-speed curve for the gearbox torque acting through the wheels. Based on $F = ma$, an "exact" expression for the car speed as a function of time can be found from the following sequence of equations:

$$F_{\text{car}} = s v_{\text{car}} + F_{\text{max}} \qquad s = \frac{-4\Gamma_{\text{max gearmotor}}}{D_{wheel}^2 \omega_{\text{max gearmotor}}} = \frac{F_{\text{max}}}{v_{\text{max}}} \qquad F_{\text{max}} = \frac{2\Gamma_{\text{max gearmotor}}}{D_{\text{wheel}}}$$

$$\frac{dv_{\text{car}}}{dt} = a_{\text{car}} = \frac{s v_{\text{car}} + F_{\text{max}}}{M} \qquad M = \frac{4n_{\text{transmission ratio}}^2\left(J_{\text{motor}} + J_{\text{transmission}}\right)}{D_{\text{wheel}}^2} + m_{\text{car}}$$

$$\int_0^{v_{car}} \frac{dv_{\text{car}}}{F_{\text{max}} + s v_{\text{car}}} = \frac{1}{M}\int_0^t dt \Rightarrow \frac{1}{s}\ln\left(\frac{s v_{\text{car}} + F_{\text{max}}}{F_{\text{max}}}\right) = \frac{t}{M}$$

$$v_{\text{car}} = \frac{F_{\text{max}}\left(e^{st/M} - 1\right)}{s} \qquad t = \frac{M}{s}\ln\left(\frac{s v_{\text{car}}}{F_{\text{max}}} + 1\right)$$

With the definition of s, which equals F_{max}/v_{max}, the car can never get all the way to v_{max} because at that point the force would be zero, so assume $v_{max\ car}$ occurs at the motor's maximum efficiency speed. Integrating velocity with respect to time to obtain an expression for the total distance traveled:

$$X_{car} = \int_0^t v_{car}dt = F_{\text{max}}\int_0^t \frac{\left(e^{st/M} - 1\right)}{s}dt$$

$$X_{car} = F_{\text{max}}\left(\frac{M}{s^2}e^{st/M} - \frac{M}{s^2} - \frac{t}{s}\right)$$

For just most robot contests, st/M is less than 1, and hence only the first three terms of a Taylor series expansion for $e^{st/M}$ are needed. This leads to a surprisingly simplified result for predicting the time it takes to reach the maximum velocity (midpoint position) of a triangular velocity profile move:

$$X_{car} = F_{\text{max}}\left[\frac{M}{s^2}\left(1 + \frac{s}{M}t + \frac{1}{2!}\left(\frac{s}{M}\right)^2 t^2 + \frac{1}{3!}\left(\frac{s}{M}\right)^3 t^3 + \ldots\right) - \frac{M}{s^2} - \frac{t}{s}\right]$$

$$X_{\text{car for st/M<<1}} \cong \frac{F_{\text{max}}t^2}{2M} \qquad t \cong \sqrt{\frac{2M X_{car}}{F_{\text{max}}}} \qquad v_{\text{max}} = \sqrt{\frac{2F_{\text{max}}X_{car}}{M}}$$

The approximate relationship is based on the condition of $st/M<<0$ which should be true for short duration motions. The surprising and delightful result is that *when the matched inertia doctrine is used to select the optimal transmission ratio, the output position is simply $at^2/2$ where the acceleration a is equal to the maximum resulting no-slip force, created by the stall torque, divided by the total equivalent system mass!* All the detailed analysis showed that a first-order estimate is pretty good for the typical task at hand, but that the "exact model" can be easily incorporated into MATLAB™ and used as shown to check the "exact" response of the system during the detailed design phase. For a start-to-stop move via a triangular velocity profile, the time to move from a standstill from one point to another and stop is thus about:

$$t \cong 2\sqrt{\frac{M X_{\text{start-to-stop move}}}{F_{\text{max}}}}$$

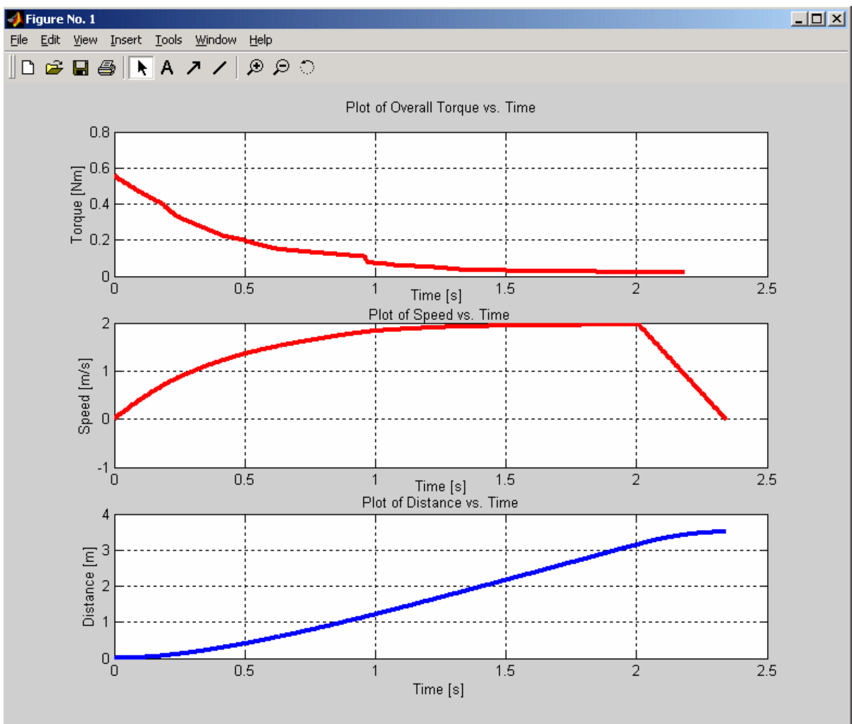
The spreadsheet *gearmotor_move.xls* incorporates these basic formulas. The MATLAB code *GROPT.m* incorporates non-linear effects which are not conducive to closed-form solutions. For example, it takes into account that system friction also helps to stop the car, it checks for wheel slippage and maximum motor velocity, it accounts for efficiency/speed at each integration step, and it does not even have to make linear torque-speed assumptions.

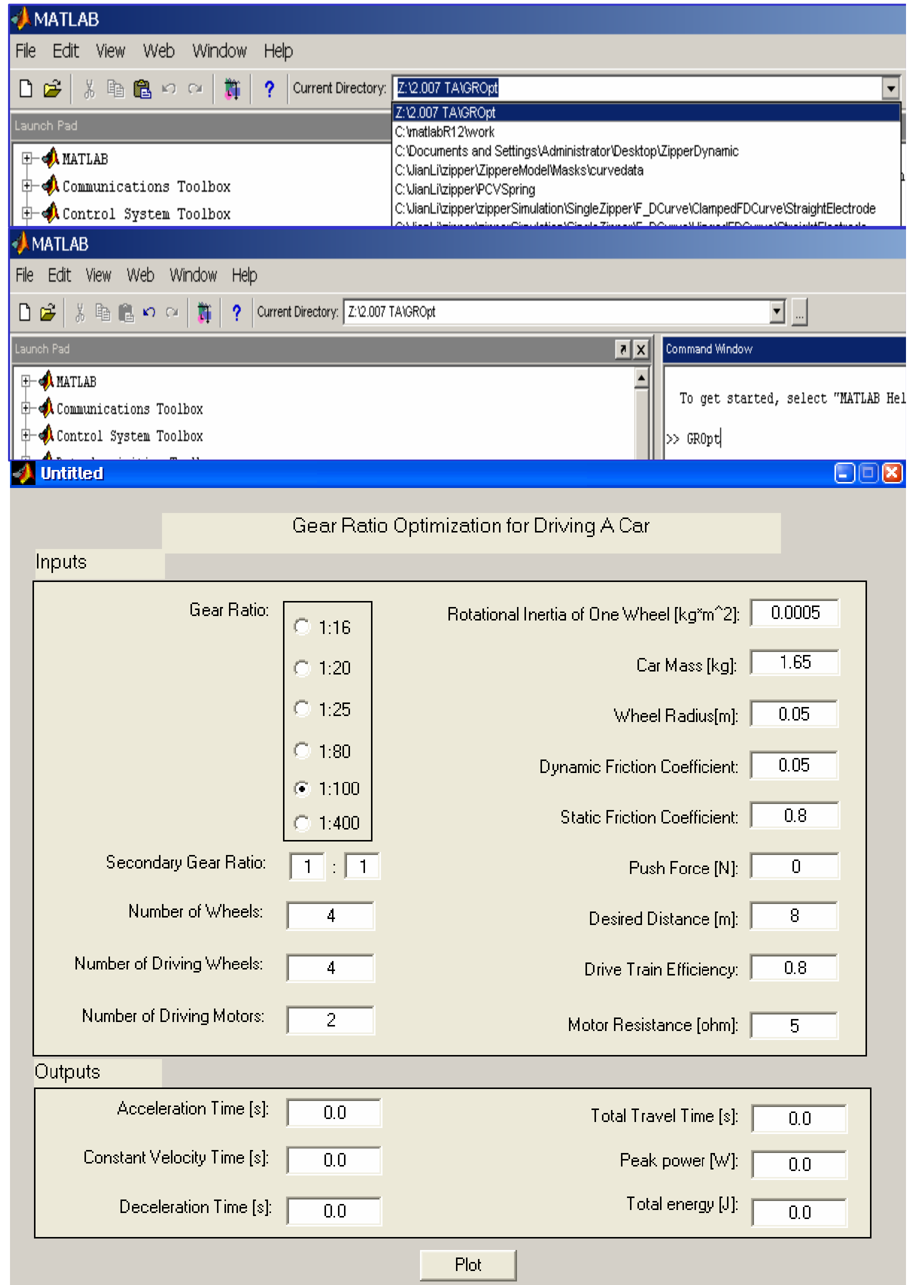
What is the appropriate analysis tool to use, the quick formula, a spreadsheet or the MATLAB code? Play with both and compare the results with respect to your machine. After you build your machine, compare the predictions to the data and enter the results into your experience database.

1. Many thanks to my graduate students and friends Jian Li and Fadi Abu-Ibrahim for their help in developing the MATLAB code and generating the gearmotor torque-speed data.

DC Brushed Motors: *Motion Simulations*

- Code written in Matlab™ enables the design engineer to integrate equations of motion step-by-step and incorporate special conditions and nonlinearities
  - This is most appropriate for the detailed design phase, particularly for critical components or systems, or high-volume products
- If sensor feedback is used for closed-loop control, motion simulations are indispensable





Gearmotors: *Shaft Loading & Wire Strain Relief*

There are four main failure modes for gearmotors: *over-torque, burn-out, shaft/bearing,* and *wiring failures*. Each of these failure modes commonly happens in robot design contests, as well as in consumer and commercial products when the design engineer does not pay attention to fundamental principles. Nevertheless, sometimes failure will occur due to unexpected loadings, such as crashes, and then it is important that the gearmotor be mounted in a simple modular manner for rapid replacement.

The output torque of the gearbox is a product of the efficiency, the transmission ratio, and the motor torque. The holding torque, however, is much higher because the inefficiency acts like a brake:

$$\Gamma_{output} = \eta \Gamma_{input} \qquad \Gamma_{holding} \approx \frac{\Gamma_{input}}{\eta}$$

Over-torque happens not from the motor trying too hard to cause motion, a well-designed gearbox should not allow the motor to damage it, but rather when too large a torque is applied to the gearbox. This can happen if a large external torque impulse occurs, such as from a crash. When an impulse is applied to the shaft output, such as from a collision, all of the gears' inertias come into play. The higher the gear ratio, the higher the apparent inertia, and the greater the chance of damage. The result is that the final stage gear teeth experience the highest loads and are most likely to break. A strategy to prevent this is to use a break-away or shear coupling; however, this can also result in the apparent output torque being way too low. In the case of the gearboxes shown on page 7-14, the protection system is removed by a small modification. This is fine for short-life applications, but would not be acceptable for a consumer product.

Burn-out is caused when the motor exceeds its thermal duty cycle. For example, if two robots are stalled pushing against each other, their motors will be drawing full-current while not turning. The temperature of the motor will rise, and the high current acting across the brushes can cause arcing. Soon, the temperature of the motor will rise to the point where the insulation on the wire that makes up the windings will break down and cause shorts. The motor has thus become "fried". The way to avoid this problem is do not stall the motors! If you are involved in a pushing contest with someone, and neither robot can advance, back off on the power, and due to the holding torque effect mentioned above, you will be able to stand your ground with far less power! Meanwhile, your opponent will fry their motors or drain their battery!

Shaft/bearing failures occur because novice engineers often see a shaft sticking out of a gearbox and somehow get the impression that it should be able to handle any loads applied to it. WRONG! the output shaft really is intended to only transmit torque to another shaft, which would be properly supported by bearings to handle the radial loads on the system. Far too many engineers have designed products, or robots for design contests, where they attach a lever or a wheel to a gearbox's output shaft. The resultant radial force caused by external loads, or the resultant radial load generated by gear teeth, cause the gearmotor's output stage's bearings to be overloaded and fail (see page 6-25).

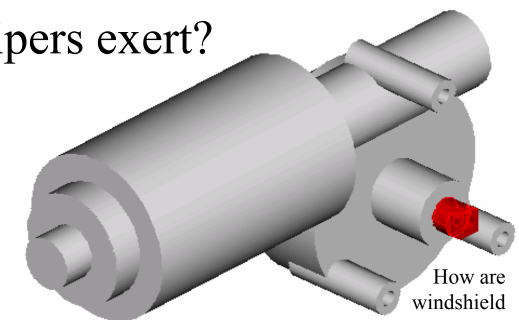
Look at the cross section through the motor gearbox, and you can see that the output stage shaft is supported by a sleeve bearing that is maybe 2 shaft diameters in length. The picture shows a purple wheel attached with the hub facing the gearmotor. This is incorrect for 2 reasons: 1) The pin through the hub transfers torque well, but the radial loads are now passing through a section of the shaft with a hole in it, and this causes a stress concentration and low-cycle fatigue. The wheel/hub should be turned around so the hub is on the outside. 2) The radial loads on the wheel are now applied about 2 shaft diameters from the bearing, and this increases the bearing loads. Again, turn the wheel around (reciprocity). Just because a company shows a picture of an application, drawn by an artist, does not mean the engineers carefully checked the packaging! *Caveat Emptor!* Note the other picture which shows how gears can be mounted so the radial loads and the gear-tooth separation forces can be made to essentially pass directly through the shaft-support bearings.

Wiring failures are commonly caused by loose wires which are accidently pulled which rips off the motor leads. A strain relief is a loop of wire anchored at two points before the wire's point of attachment. If the wire is snagged, one attachment point and the loop give without forces being transmitted to the wire-to-motor lead attachment. All wires and hoses should have this form of double-attachment point strain relief next to any connection!

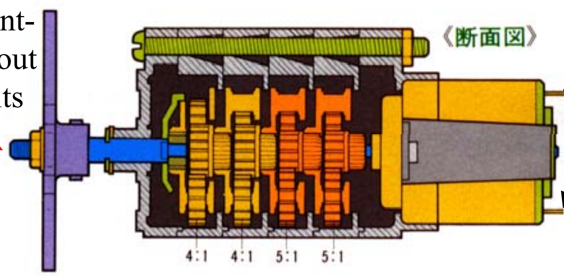
Check your shaft mountings and your wiring! Even when building your robot, wires can be pulled and shafts overloaded, so take care to do things right at all times!

Gearmotors: *Shaft Loading & Wire Strain Relief*

- Consider a windshield wiper motor, how much of a load can the wipers exert?
  - The output shaft is 0.47” in diameter. It is a worm drive system, with the output shaft bearings about 3 shaft diameters apart.
- Consider gearmotors for robot design contests, how large of a radial load can the output shaft support?
  - Can gears be mounted such that the radial forces are nearly over the front support bearing?
  - Can gears be effectively used as a coupling…

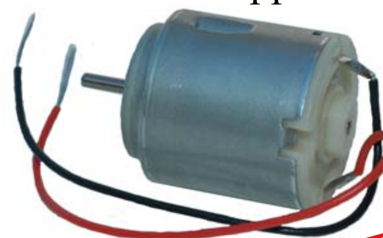


How are windshield wiper blades attached?

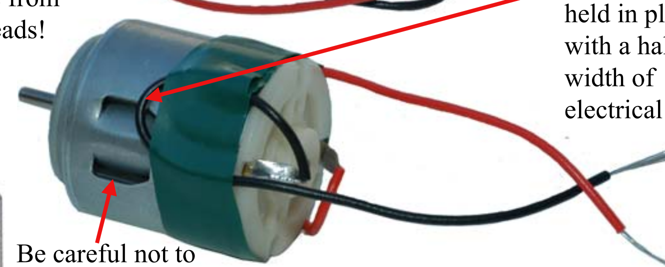


What does Saint-Venant say about this shaft and its load?!

SMALL DELICATE LEADS to which THIN wires are to be soldered and then wrapped and taped to the motor to prevent external loads from being applied to the leads!

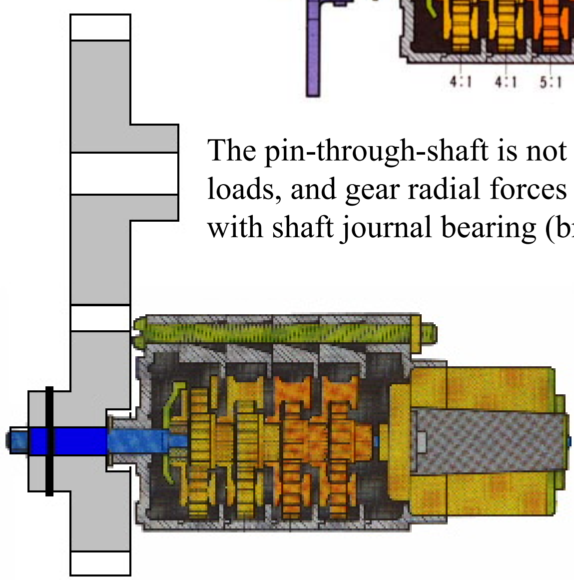


Motor as received

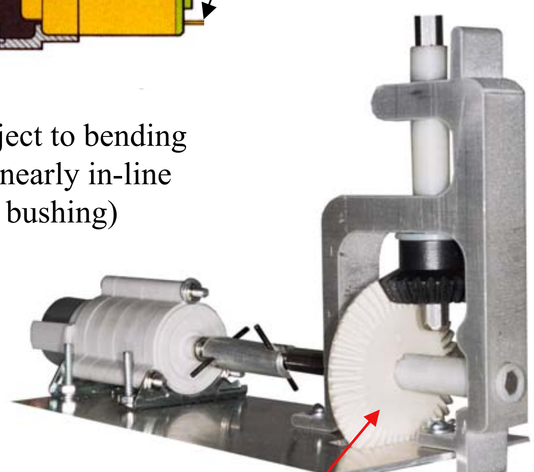


Wire makes a loop, and is held in place with a half-width of electrical tape

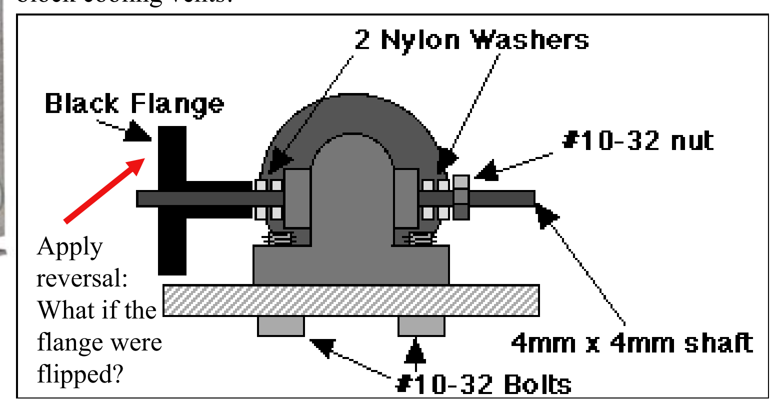
Be careful not to block cooling vents!



The pin-through-shaft is not subject to bending loads, and gear radial forces are nearly in-line with shaft journal bearing (brass bushing)



Should the big gear really be driving the little gear?



Case Study: *A CD Drive*

Designers of CD drives are under incredible pressure to produce high precision low-cost devices, so they need to rely heavily on FUNdaMENTAL principles. Recall the discussion on page 6-9 about the mechanics of a gear-driven leadscrew for a computer's CD drive. An electric motor's output shaft has a spur gear pressed onto it. The spur gear (1) drives a spur gear (2) that is molded integral with a helical gear (3). This helical gear engages another helical gear (4) that is attached to a leadscrew. The leadscrew is axially constrained by a spring that pushes it so its helical gear always produces a torque to keep the gear train preloaded; hence eliminating backlash. What is the relation that describes the force generated on the stage by the motor inertia with torque Γ, gear ratio n, leadscrew lead ℓ, and drivetrain efficiencies that cause the drive's read head assembly to move with the required precision?

$$F_{stage} = \left(\frac{2\pi\eta_{leadscrew}}{\ell_{\text{leadscrew lead}}} \right) \left(\eta_{gears} n_{\text{transmission ratio}} \Gamma_{motor} \right)$$

A controls engineer would create a model that includes the mass of the stage, and the inertia of the leadscrew, gears and motor rotor. The model would be allow the engineer to create a control algorithm that the CD drive's control computer would use to position the stage with a resolution of motion that would still allow the Lorentz Force actuated *Optical Pickup Unit* (OPU) to rapidly move to correct for position errors in the large stage.

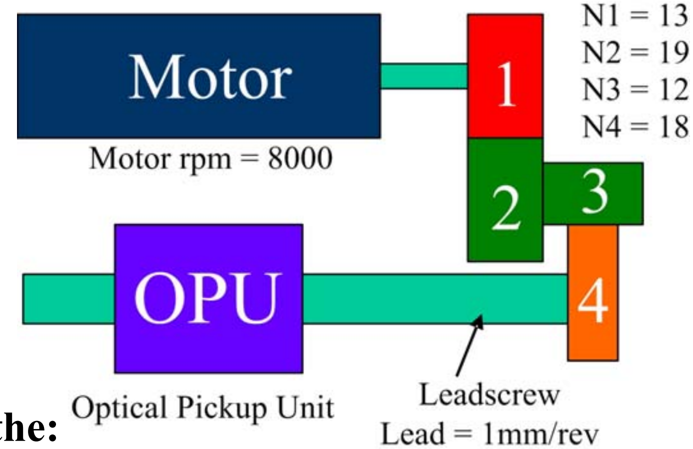
The close-up picture of the OPU shows a flexural-bearing supported stage actuated by a Lorentz force actuator: The OPU is supported by small wires which also carry current into the coils. The current goes into a coil from the right wire/flexure and then to the other coil before exiting out the left wire/ flexure. The magnetic circuit (purple curved arrows) crosses the coils ((+) current into page) so that all of the coil wires produce a force (green arrows) in the same direction. This allows the OPU to move with a servo bandwidth much faster than the leadscrew driven stage that carries the OPU. As a result, small non-linear motions in the leadscrew driven macro stage are easily compensated for by small high speed motions in the micro stage. This type of system is often called a *master/slave* or *master/follower* stage system and it is often used in many different types of precision machines.

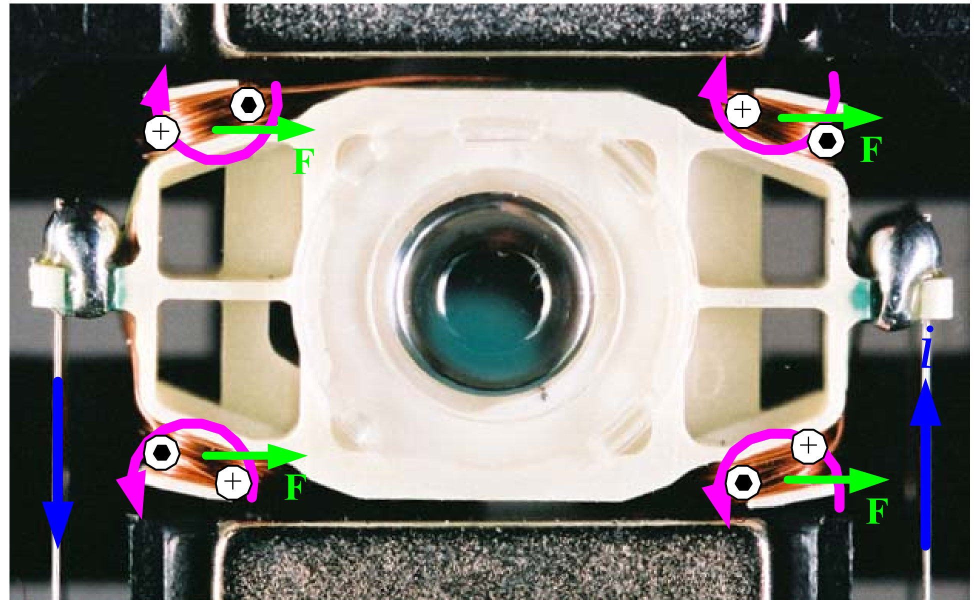
To begin the design of the CD Drive's main stage, an engineer would first do a quick assessment to ensure that the force that can be exerted on the stage by the motor at its peak operating efficiency torque is capable of accelerating the stage at the desired acceleration. *Motor_gearbox_leadscrew.xls* shows the type of calculations needed. Note that the actual motor/transmission system inertia is not known, but can be assumed to be equal to the stage mass by the matched inertia doctrine. Thus this spreadsheet acts as a first-order estimator to determine system feasability.

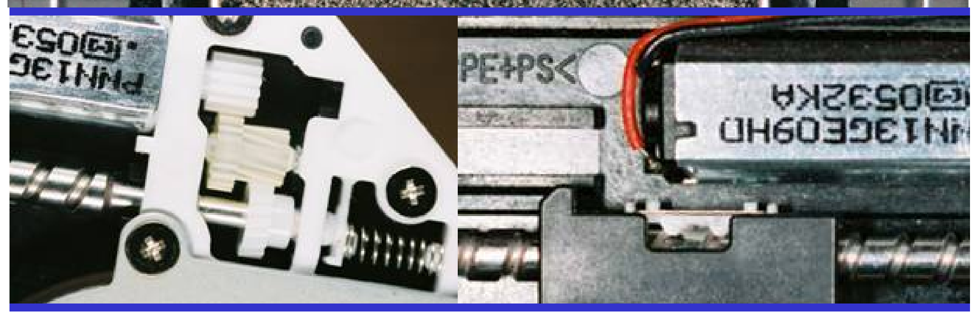
Leadscrew		
leadscrew lead, L (mm, m)	1	**0.001**
Coefficient of friction, mu	**0.05**	
Screw pitch diameter, dscrew (mm, m)	**1**	**1.00E-03**
Thrust bearing diameter, dthrust (mm, m)	**0.5**	**5.00E-04**
Thread angle (deg), alpha (rad)	**30**	**0.524**
Beta	**1**	
Backdriveable?	**YES**	
Thread efficiency, etathread	**83%**	
Thurst bearing efficiency, etathrust	**94%**	
Total leadscrew system efficiency, etascrew	**78%**	
Gears		
Gear 1, N_1	**13**	
Gear 2, N_2	**19**	
Gear 3, N_3	**12**	
Gear 4, N_4	**18**	
Efficiency per stage, etastage	**95%**	
Geartrain efficiency, etagears	**90%**	
Geartrain ratio, n	**2.19**	
System		
Force gain: force = GAIN*(Motor Torque), gain	**10744**	
Force on stage, Fstage (N)	**2.15**	
Stage acceleration (assume matched inertia doctrine: motor/tranny inertia = stage inertia), (m/s, g's)	**43.0**	**4.4**

Use the spreadsheet to determine the optimal transmission ratio for a motor-gearbox-leadscrew-stage system, e.g., on your robot. Play with it and compare the results to your initial calculations. Is your design still feasible or do you have to make changes, such as reducing the system mass?

Case Study: A *CD Drive*

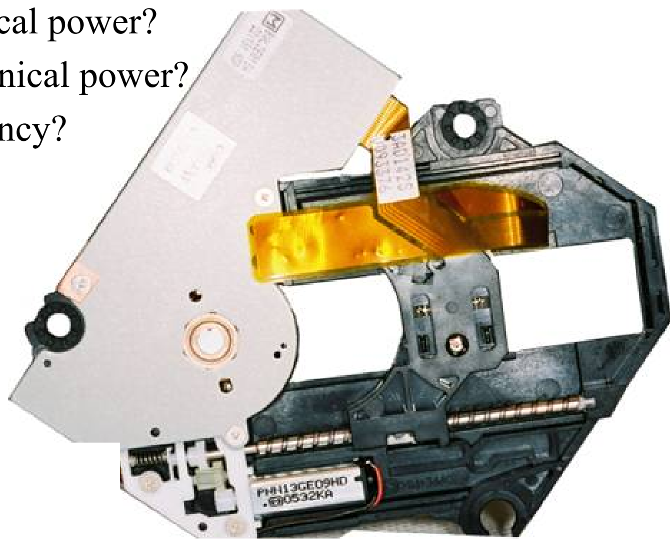




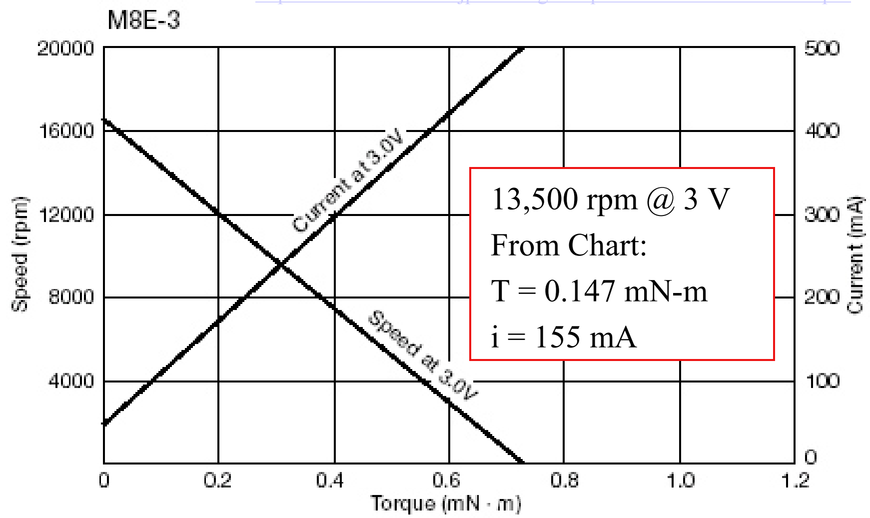


- **What is the:**
  - Gear Train Ratio?
  - Median output velocity?
  - Output torque at this velocity?
  - Linear speed of the OPU?
  - Force available to overcome friction?
  - Electrical power?
  - Mechanical power?
  - Efficiency?

Special thanks to Dr. Patrick Willoughby for creating this case-study



Motor data from Mitsumi at http://www.mitsumi.co.jp/Catalog/compo/motorav/m8e/3/text01e.pdf



Energy Supplies

In order for your machine to move, forces need to be applied, which is a direct consequence of Newton's laws. In addition, the second law of thermodynamics tells us that entropy is always increasing. The goal in a robot design contest is to make sure that your machine has more than just enough energy to accomplish all the desired tasks.

In fact, when designing a machine, such as a robot for a design contest, the designer ideally has kept track of the different motions and forces required, and this forms the basis for a *power budget*. A power budget keeps track of what power is needed at what time and for how long during a machine's operating cycle. Once the power budget has been created, the designer can then select the appropriate energy storage element to provide the required power.

Often, the available energy storage elements have been given, such as in a robot design contest, and these very elements may be what catalyzed the design idea in the first place. One wants to avoid developing a great design, only to find out that one does not have 1.21 gigawatts of power available! So which comes first, the battery or the motor? The spring or the projectile? The answer is probably "both": Apply *Maudslay's Maxims* and perhaps by listening to the Rolling Stones, (see page 1-4) you should be able to develop a design that functions admirably.

Robots can be powered by umbilicals, batteries, compressed gas bottles, springs, internal combustion engines, fuel cells, solar cells, gravity (its free!)... Starting with umbilicals, one might think that they can provide infinite cosmic power through a little cable. However, the smaller the cable, the less power it can transmit. For example, a small diameter air hose connected to a large diameter pneumatic piston will move the piston far more slowly than if the piston were connected to a large diameter hose. The electrical and fluid resistances of an umbilical cable are part of the circuit itself.

If fact, it is not only the electrical or fluid resistance of the umbilical lines that is an issue, it is the mechanical stiffness and resistance of physical cable that can retard or impede the motion of a robot or one of it's moving axes. Moreover, even in the world's most sophisticated machines, such as photolithography machines[1] used to expose silicon wafers, the design of the cable handling system often is the deciding factor in selecting a "best" design. In product after product, when the wires are not properly routed and controlled using well-designed umbilicals, the product will fail no matter how brilliant and clever the rest of the design. Hence for many robot design contests, umbilicals connecting the machines to the outside world are being phased out in favor of batteries, springs, and compressed air bottles (and gravity!).

Batteries store electrical energy, and they can be disposable or rechargable. In addition to considering the short term cost (buying them), and the long term environmental cost (landfill mass vs. heavy metals), practical issues such as internal resistance limiting current draw, size, and recharging characteristics must be considered. In general, one can look to the past to see what could be used for the present: What types of batteries are used by what types of machines that are turned on and off a lot, and are required to deliver large bursts of power, be rugged and be economical? The answer in many cases are NiCad batteries that power cordless products such as drills.

Compressed air bottles require careful regulation because of the potential for an explosion should the bottle be pierced or overpressured. One can simply pressurize a PET soda bottle and cover it with a strong cloth covering to prevent it from being scratched and weakened; however, any pressurized container legally is supposed to be certified by an appropriate government agency. Disposable CO_2 cylinders, such as those used in paintball guns or soda bottles, can be used, but they can be expensive. Whenever pressurized air is used, it should be used according to established safety standards!

Mechanical springs store energy by elastic deformation, typically of metals, although elastomers (rubber) are sometimes used. Springs can be used to store and rapidly release large amounts of energy, if the trigger is properly designed!

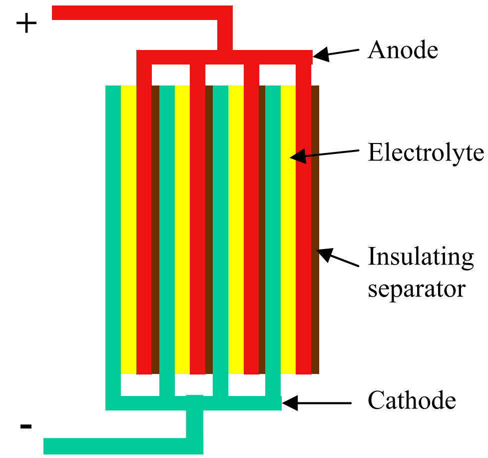
Review the different types of energy storage and supply elements in your kit. Make a table showing how much energy they can store, and compare it to some of the functions you want your robot to accomplish.

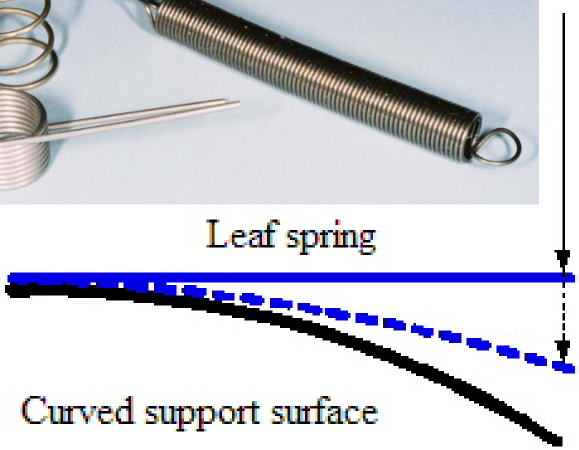
1. See for example www.asml.com and US Patent 6,262,796

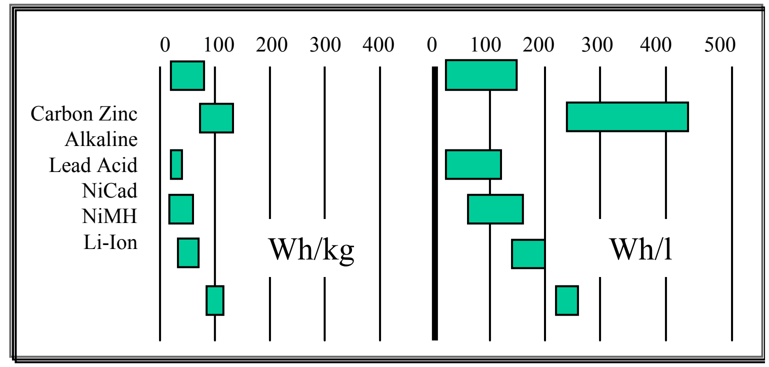
Energy Supplies

Linus Park created an awesome walker for 2.007 2000 MechaImpactAgeddon! But like all students that year, he had to deal with the **umbilical**!

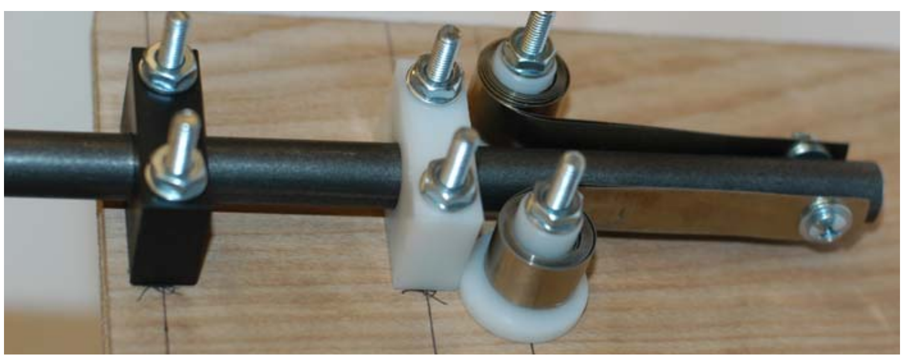
- Umbilicals
- Batteries
- Springs
- Gas cylinders
- *(Don't forget the gravity!)*







Batteries store LOTS of energy and a wiring short can release it!



Energy Supplies: *Batteries*

Batteries store electrical energy by the mechanism of electrochemical potential which converts chemical energy into electrical energy. The anode (positive terminal) is the material that gives off positive ions that flow through an electrolyte to the cathode (negative terminal)[1]. An anode/cathode pair is called a "cell", and a battery is a set of cells. Different batteries have different characteristics as shown in the table. A seemingly detrimental characteristic, such as a high internal resistance, which limits the current a battery can provide, can also give the battery a good characteristic, such as long shelf life. Thus one should at least consider the different characteristics shown in the table when selecting a battery.

All batteries have some internal resistance, and this is what limits the current that can be drawn from them. The table shows several different types of AA batteries. Common alkaline batteries, which are called alkaline because of the potassium hydroxide (KOH) electrolyte, have relatively high internal resistance. The table of alkaline battery properties from a major manufacturer shows how they typically have 150 mΩ resistance per cell, but this also gives them a long shelf life. Each battery has a 1.5 volt potential, so if 5 batteries are used in series, then together they have 0.75 ohms resistance. When connected to a motor with 3 ohms resistance, the system looks like a simple voltage source in series with the battery's internal resistance and the resistance of the motor windings, and the total resistance of the circuit is 3.75 ohms. At stall, the motor is not turning and there is no back-emf being generated, so the voltage drop across the motor is:

$$V_{motor} = V_{battery}\frac{R_{motor}}{R_{motor}+R_{battery}} = 7.5\frac{3}{0.75+3} = 6 \text{ volts}$$

As the motor brushes wear-in to make better contact with the commutator, the motor resistance drops to as low as 2 ohms. Would the designer be better off using rechargeable NiCad batteries? *Batteries.xls* explores some of the scenarios. It is shown that for high current critical situations, the NiCad batteries with their lower internal resistance, even though they have lower voltage per battery, are the best choice:

	Alkaline	NiCad	Alkaline	NiCad
Number of cells	5	5	5	5
Resistance per cell (ohm)	**0.15**	**0.02**	**0.15**	**0.02**
Volts per cell	**1.50**	**1.20**	**1.5**	**1.2**
Motor resistance (ohm)	**3.00**	**3.00**	**2.00**	**2.00**
Number of motors in parallel	1	1	1	1
Total circuit resistance (ohm)	**3.75**	**3.10**	**2.75**	**2.10**
Voltage drop across motors	**6.0**	**5.8**	**5.5**	**5.7**
Current through motor windings	**2.0**	**1.9**	**2.7**	**2.9**

In either case, as the motor speed increases, the rotor spinning in the magnetic field acts like a generator, producing a voltage that gradually reduces the potential across the resistors. Eventually, equilibrium is reached, no current flows, no torque is generated, and the motor reaches its maximum speed.

It is common for a set of NiCad batteries to be used in a robot design contest, although they can be comparatively expensive; and after the contest, what do you do with them? Students also sometimes accidently short the batteries, which can greatly reduce their life and leave next year's recipient of the batteries unhappy. In addition, NiCad batteries can have a memory effect, and gases can be generated when recharging them which can degrade their performance. Completely draining the batteries and then charging them slowly helps to maintain their performance capabilities, although a good quality charger will typically take care of these issues.

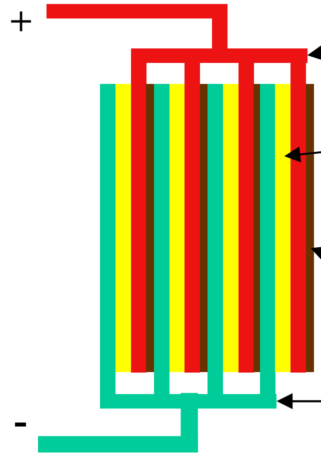
Batteries from a consumer product, such as a video camera, often have internal logic that detects current or voltage spikes and triggers an internal circuit breaker which can only be shut off if the battery is returned to its charger. Cordless drill batteries, on the other hand, are designed to handle large loads and are much more robust. They can also be used in a drill afterwards which is one of the most handy tools in an engineer's toolbox.

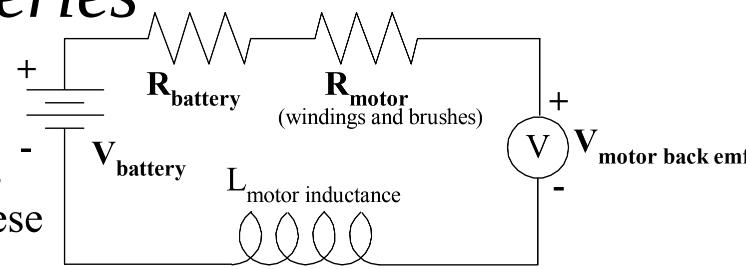
What is the energy storage capacity of your batteries? How well do they respond to a heavy load? Can you design and run a simple test for your batteries and assess their potential? How might their performance change your design concepts and operational strategy?

1. To remember which is which, think of the anode is like "add" node and "add" is positive, so the anode is the positive terminal.

Many thanks to Mitchell Weiss for help with this battery section

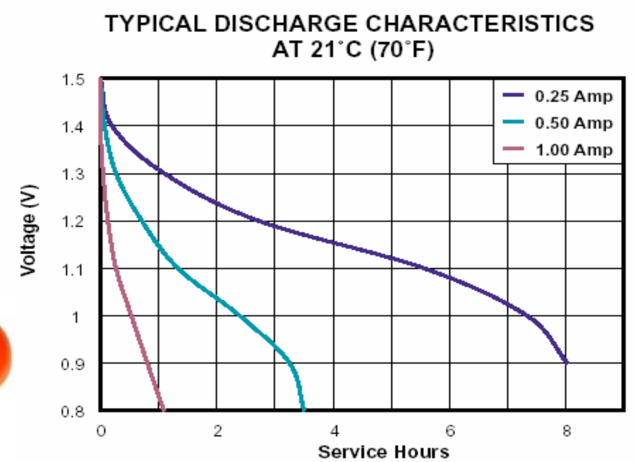
Energy Supplies: *Batteries*



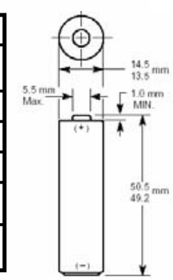


- Primary
  - Use once & dispose: Carbon-Zinc, Mercury-Oxide, Alkaline-Manganese
- Secondary
  - Rechargeable: Lead-Acid, Nickel-Cadmium, Nickel-Metal-Hydride, Lithium-Ion
- Fuel Cells
  - Consumable active material

Type	+ve	E'lyte	-ve	V	% dis	I-out
Carbon-Zinc	MnO_2	$NH_4ClZnCl_2$	Zn	1.5	10/yr	Lo
Alkaline	MnO_2	KOH	Zn	1.5	7/yr	Lo
Lead-Acid	PbO_2	H_2SO_4	Pb	2	20/mo	Med
NiCad	NiOOH	KOH	Cd	1.2	20/mo	Hi
NiMH	NiOOH	KOH	Fe	1.2	20/mo	Med



Nominal Voltage:	1.5 V
Nominal Internal Impedance:	181 m-ohm @ 1kHz
Average Weight:	23.8 gm (0.84 oz.)
Volume:	8.4 cm^3 (0.51 in.^{3})
Terminals:	Flat
Operating Temperature Range:	-20^{o}C to 54^{o}C (-4^{o}F to 130^{o}F)
ANSI: IEC:	15A LR6



Duracell AA-MN1500 battery from http://www.duracell.com/oem/Pdf/MN1500.pdf

Alessandro Volta, (1745-1827)

The ideal robot energy source: a cordless drill battery

Energy Storage: *Springs*

There are three fundamental modes of elastic energy storage used by springs: tension/compression, bending, and torsion. Tensile/compressive strain energy storage is more rare because by its nature, it will not generate large displacements. Leaf springs store energy by bending a beam. Torsion springs store energy by twisting a shaft. Coil springs actually store energy by a combination of shear and torsion. Many springs have a linear force/deflection response, where the force equals the product of the spring constant and the deflection. For tensile/compressive springs with cross sectional area A, length L and modulus of elasticity E:

$$F = kx \qquad k = \frac{AE}{L} \qquad \sigma_{\text{max stress}} = \frac{F}{A}$$

A simple cantilever leaf (bending) spring of constant width and thickness stores most of the energy at the base of the cantilever, and this is where it will break. Look at the finite element model of the tapered cantilever beam used as a MEMS spring. A tapered beam leaf spring stores more strain energy because the material is equally strained all along its length. The spreadsheets *Tapered_thickness_leaf_spring.xls* and *Tapered_width_leaf_spring.xls* allow you to play leaf spring design. Note that to be an effective spring, the spring material has to have a high yield strength. What trends can you spot in the following expressions for leaf (bending beam) springs:

Straight:

$$k = \frac{3EI}{L^3} \qquad \sigma_{\text{bending stress}} = \frac{6FL}{bt^2}$$

Width tapered:

$$k = \frac{Et^3m^3}{6\left(-2Lam + L^2m^2 + 2a^2\ln\left(1+mL/a\right)\right)} \qquad m = \frac{b-a}{L}$$

Thickness tapered:

$$k = \frac{bE}{12\left(\frac{t_{end}-t_{base}}{2m^3t_{base}} - \frac{\ln\left(t_{end}/t_{base}\right)}{m^3} - \frac{L\left(2t_{base}-t_{end}\right)}{2m^2t_{base}^2}\right)} \qquad m = \frac{t_{base}-t_{end}}{L}$$

The equations are all linear in force. For the spring constant to increase with deflection, a curved support surface can be placed below the spring as shown. One could even imagine a tapered beam with this type of support. In other words, imagination and math are a powerful combination!

Torsion bar springs can store significant amounts of energy, but they do not have the L^3 factor of bending beam (leaf) springs so the need to be long. The principal challenges are to anchor the ends so as to not create a stress concentration and to avoid torsional buckling and failure. *Torsion_rod_spring.xls* uses the following equations to help design torsion rod springs:

$$\Gamma = k\phi \qquad k = \frac{GI_P}{L} \qquad \tau_{\text{max shear stress}} = \frac{D\Gamma}{2I_P}$$

$$\Gamma_{buckle} = \frac{2\pi EI_P}{L} \qquad I_P = \frac{\pi D^4}{32} \qquad G = \frac{E}{2(1+\eta)}$$

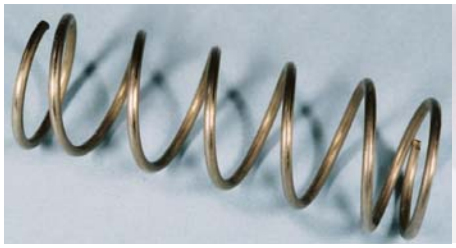
For a helical spring, the *spring index*, *stress concentration*, and *shear stress* due to a force F acting on the spring with outer diameter D and wire diameter d are given by:

$$C = \frac{D}{d} \qquad K_s = \frac{2C+1}{2C} \qquad \tau = K_s\frac{8FD}{\pi d^3}$$

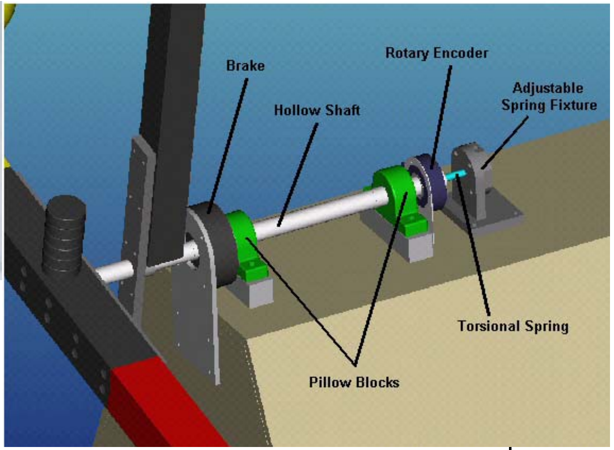
A coil spring is ideally loaded through its center, so the coils are loaded by torsion and shear, and the *deflection* and *spring constant* can be determined using Castigliano's energy method (see page 8-21), where $\Gamma = FD/2$, $L = \pi DN$, $J = \pi d^4/32$, $A = \pi d^2/4$, and N is the number of coils:

$$\delta = \frac{\partial}{\partial F}\left(\frac{\Gamma^2 L}{2GJ} + \frac{F^2 L}{2GA}\right) = \frac{4FDN}{d^2}\left(\frac{2D^2}{d^2}+1\right) \approx \frac{8FD^3N}{Gd^4} \qquad k = \frac{Gd^4}{8D^3N}$$

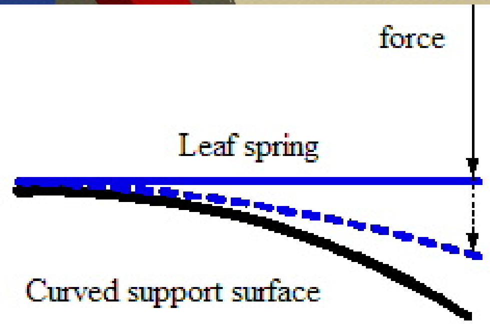
How might you use springs in your robot to reduce the number of actuators required? How much energy can you store in the springs that come in your robot kit? How does this energy compare to the batteries? Which energy storage element provides greater power (energy/time)?



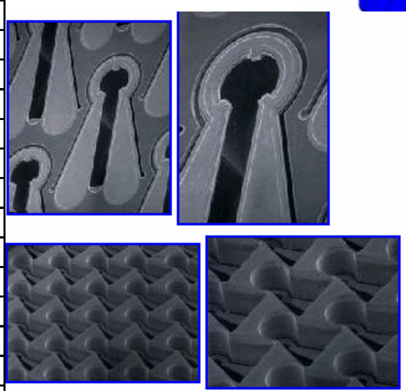
Springs



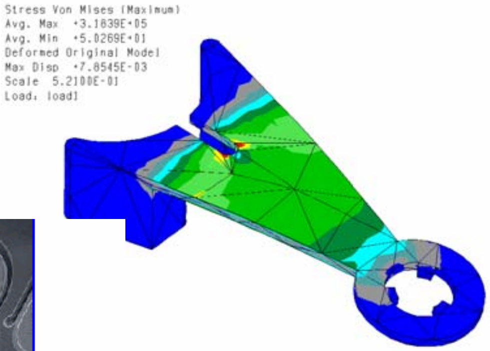
- Springs are extremely useful:
  - Small linear extension and torsional springs are often used in triggers and doors
  - Constant force springs are a *serious* energy source!
- Springs are made from hard steel, and may be difficult to modify without weakening them (be careful!)
  - Use the mounting features that are integral to the spring!
- You can make your own springs with careful design of beams or torsion rods



Tapered_thickness_leaf_spring.xls		
To determine stress, deflection, & spring constant of a tapered-thickness constant-width beam		
Last modified 8/28/03 by Alex Slocum		
Enters numbers in **BOLD,** Results in **RED**		
Force, F (N, grams)	**0.5**	51
Modulus, E (N/mm^2)	**1.50E+05**	
Width, b (mm)	**0.5**	
Thickness at end, te (mm)	**0.1**	
Thickness at base, tb (mm)	**0.2**	
Length, L (mm)	**5**	
distance along beam (x=0=end), x (mm)	**0**	
Thickness slope, m	**0.02**	
constant, c_1	**9375**	
Max stress at base, maxs (N/mm^2)	**750**	
Max strain at base (%)	**1%**	
Slope (radians)	**-0.250**	
Deflection, defl (mm)	**0.681**	
Spring constant, k (N/mm)	**0.734**	
Comparative straight beam defl	**0.417**	
tapered/straight beam deflection	**1.64**	



MEMS silicon microcantilevers



Tapered_width_leaf_spring.xls		
To determine stress, deflection, & spring constant of a constant-thickness tapered-width beam		
Last modified 8/28/03 by Alex Slocum		
Enters numbers in **BOLD,** Results in **RED**		
Force, F (N, grams)	**0.05**	5.1
Modulus, E (N/mm^2)	**1.50E+05**	
Thickness, t (mm)	**0.030**	
Length, L (mm)	**0.5**	
Width at tip, a (mm)	**0.050**	
Width at base, b (mm)	**0.200**	
resulting slope, ms	**0.300**	
I/c max, Ioc (mm^3)	**3.00E-05**	
Moment, M (N-mm)	**0.025**	
Max stress, maxs (N/mm^2)	**833**	
Max strain (%)	**0.56%**	
deflection at end, defl (mm)	**0.040**	
Spring constant, k (N/mm)	**1.263**	
Comparative straight beam defl	**0.031**	
tapered/straight beam deflection	**1.28**	

Springs: *Applications*

Springs have many uses, such as helping to accelerate the motion of a robot component, like an arm or projectile. However, springs in their free state have no energy stored, so they have to be *preloaded* (deformed or primed) in order to initially store any energy. This requires one end to be attached to the machine structure, and the other end to be attached to the component to be moved. In general, a goal is to minimize the mass of the object to be moved so acceleration can be maximized. The amount of extension one puts into a spring so that in its mounted state it is exerting an initial force is called the *pre-load displacement.*

Consider the use of a common extension spring, whose force is proportional to the extension, to launch a projectile. How fast will the projectile be moving when the spring is no longer stretched and it has given up all its energy to the mass? One could start with the relation $F = kx$ and recognize that $F = ma$ and $a = d^2x/dt^2$ and proceed to solve the differential equation. This would give an expression for the velocity as a function of the spring's extension; however, at the concept stage, one just needs to know the velocity of the mass as it leaves the launching system. The appropriate level of analysis at this stage is to equate the potential energy stored in the spring with the kinetic energy of the mass as it leaves the launcher:

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

This shows that the velocity of the mass is proportional to the product of the preload displacement x of the spring and the frequency at which the mass would oscillate where it attached to the spring!

Although this simple model is appropriate for the initial conceptual design phase, it does not take into account friction in the launching system. How can friction be accounted for? If the primary friction force is simple coulomb friction, where the friction force is equal to the product of the normal force and the coefficient of friction, then velocity is not a factor. The friction force, such as $F_{normal\ force}\mu$ acting over the distance traveled can then be subtracted from the kinetic energy of the spring:

$$\tfrac{1}{2}kx^2 - F_{friction}x = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{kx^2 - 2F_{friction}x}{m}}$$

If the spring force has a component that causes friction, one could predict what the effect would be; on the other hand, one could realize that this takes time and the prediction might be too dependant on how the machine was built. Hopefully this thought is combined with the lesson of *Centers-of-Action* from page 3-26 so the spring force is applied through the centers of mass, friction and stiffness!

Of particular interest are constant force springs, which are made from a coil of flat steel. the spring fits over a post, but it does not need to be prevented from rotating at its center. The outer free end of the spring is then attached to the object to be moved. As shown, there are many different ways to mount constant force springs, but symmetry is often key to avoiding parasitic forces which give rise to friction forces. The energy stored in a spring with constant force is just the product of the preload displacement x and the constant force F it provides. Hence the velocity of an object propelled by a constant force spring is found from:

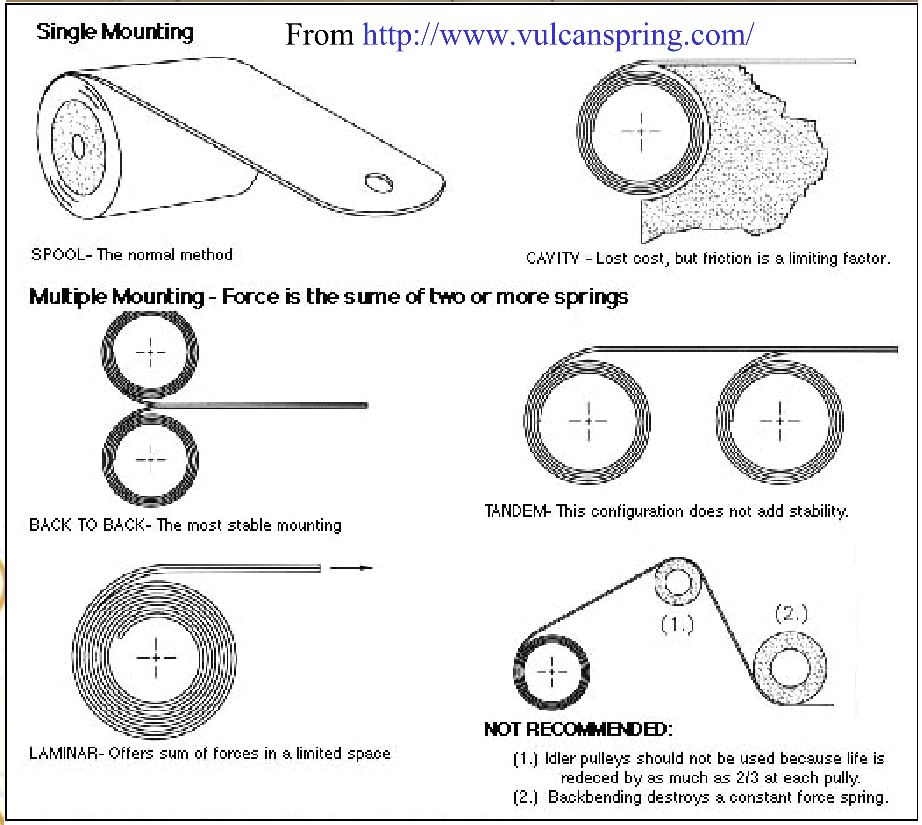
$$Fx = \tfrac{1}{2}\left(m_{projectile} + m_{carrier}\right)v^2 + F_{friction}x \Rightarrow v = \sqrt{\frac{2x\left(F - F_{friction}\right)}{m_{projectile} + m_{carrier}}}$$

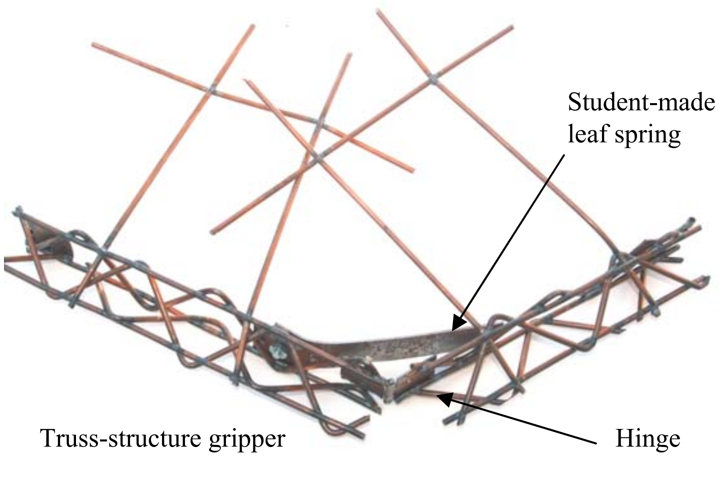
Springs are often damaged when they are overextended. Once overextended, the helix angle, in the case of a coil spring, is increased significantly, and the spring rate decreases. Constant force springs, on the other hand, are meant to be extended, but if they are pulled too far, their end can come off the mounting post and then suddenly and dangerously roll up in your face! Always be extremely careful when working with springs!

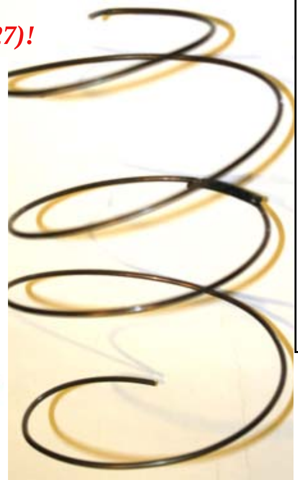
Are you planning on using a projectile? How far can you launch it? At what angle should it initially be projected to get maximum range? Does 45 degrees sound correct? Dust off the college freshman physics book!

Springs: *Applications*

- The MIT 2.007 kit contains 4 constant force springs from Vulcan Spring Corp:
  - 0.75" ID, 0.475" x 0.009" thick 301 Stainless Steel, with a pull force of 2.6 lbf [11.6 N]
- Coil springs: Check the kit for various types
  - Devanie Dufour '08 showed that you can make a very nice large diameter spring from welding rod!
    - See *Coil_spring.xls*
- Constant force springs make excellent preload devices, launchers….
  - The should be mounted ideally without modifying their structure (drilling, bending) because they could then easily break!
- ***Remember Centers of Action*** ***(page 3-27)!***







Pneumatic Systems

Pneumatic systems can use pressurized air from an umbilical line or storage tank to exert a force on a piston that is proportional to the pressures acting on each side of a piston. Pneumatic cylinders (pistons), therefore, are a practice manifestation of Pascal's principle: *Pressure applied to a enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.* Just like an electric circuit, recall *batteries.xls*, if the diameter of the line is too small, and it is too long in comparison to the volume of the cylinder, then the piston will move slowly, and the maximum potential force that the piston can exert will take some time to realize. However, it is difficult to predict the fluid resistance through corners and valves, thus handbooks, or a simple bench level experiment, are needed to determine just how large a pneumatic line should be.

Pneumatic lines are often carried by *umbilicals* which can also be used to carry electric wires; however, they can get in the way of a vehicle and cause entanglement and drag forces. If the machine is to carry a compressed gas bottle, extreme care must be taken. Legally, in many countries, a container cannot be used to store compressed gas unless it has been certified for that purpose. In addition, for safety reasons, no student should be working around pressures greater than 6 atm. (0.6 MPa, 90 psi). Consider 3 atm. of pressurized air in a 50 ml plastic syringe. It stores the energy equivalent of 2 constant force springs[1]. *Now imagine a plastic 2 liter soda bottle with 6 atmospheres of pressure. It can store the energy equivalent of about 288 of the constant force springs in the robot kit! Imagine if all that energy were suddenly released by a scratch in the soda bottle that caused it to fail and explode in your face!* The formulas for the energy stored in a pneumatic system are derived on the next page

Pneumatic cylinders, which are often just called *pistons*, can be *single acting* or *double acting*. A single acting cylinder only provides pressure to one side of a piston, and it relies on a spring force or gravity to push the piston rod back in. A double acting cylinder has pressure supply lines connected to the housing at each end, and thus can provide a differential pressure on the piston, thereby causing it to move in or out. Unless the rod extends through both sides of the piston, the area on one side will be greater than on the other, with a similar difference in the amount of force that can be created.

Rotary motion pneumatic actuators can use a piston to drive a rack that engages a pinion gear attached to a rotating shaft, or the air can act directly on a radial vane attached to a shaft. The challenge is to obtain a good seal between the vane and the actuator housing. Hence *pneumatic vane actuators* are not routinely used, and designers generally prefer to try and obtain rotary actuation through a piston-actuated linkage. Hydraulic vane actuators are more common and are used for rotary motion at the ends of crane booms.

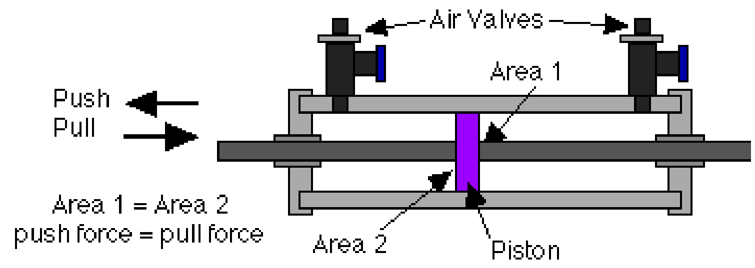
Pneumatic cylinders can be controlled by simple mechanical on/off valves or mechanical proportional valves. For robot contests, mechanically actuated valves would have to be located in the contestant's control box, and the resulting long line to the machine through the umbilical would cause serious time delays: The control valve should be as close to the actuator as possible, so electrically actuated valves ideally would be used.

Solenoid valves use a solenoid to open or close a valve so it is difficult to achieve proportional control of a pneumatic line's pressure or flow. Pneumatic systems can be controlled by pulsing on/off solenoid valves, and cylinder speed can also be regulated by flow through an adjustable orifice. With digital electronics, a valve, or an array of valves, can be turned on and off rapidly to obtain the desired speed control electronically. Force control can be even more difficult to achieve, so pneumatic cylinders in robot design contests are often used as point-to-point displacement control devices.

If air pressure and valves are available for your contest, how will you decide when to use pneumatic actuators verses electric motor or spring actuation?

1. Due to the tremendous energy storage potential that hard plastic structures in general should never be used for compressed air storage or transport, because they have a tendency to shatter. of particular danger is PVC pipe, which some people have been known to use to plumb compressed air lines. Even stoopider is the use of PVC pipe to make a compressed air "cannon" to launch things like tennis balls...PVC can shatter under high air pressure. The only pressure application for plastic structures is when they are used with liquids, because of the extremely low compressibility of liquids, they store little energy when pressurized.

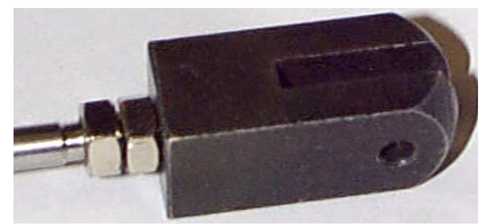
Pneumatic Systems

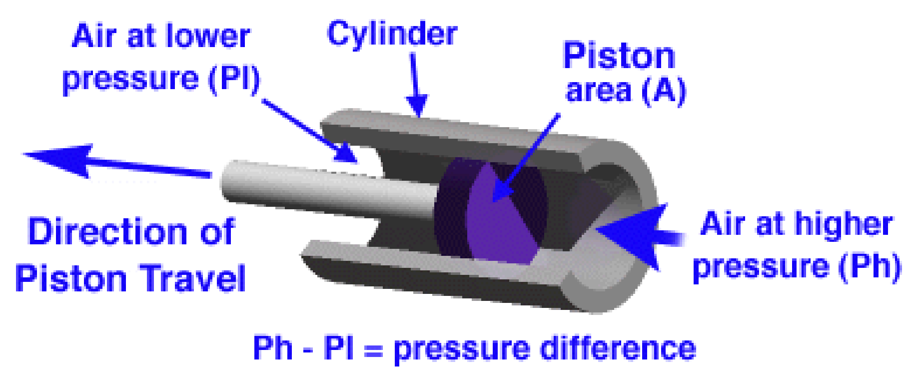


Double-Rod Double-Acting Piston

- Pneumatic (and hydraulic) cylinders let energy be piped in from a distant source
- Fluid pressure (pneumatic or hydraulic) in a cylinder acts on a movable surface (the piston) which generate a force that is then transmitted through the rod
    - Provided through an umbilical line, or from a pressurized gas bottle
    - A small solenoid valve is used to control the flow of gas
    - Incredible forces can be achieved through the use of a large area

- Pneumatic cylinders are often used when cleanliness is important
    - Double acting pistons have different push and pull areas!
    - Speed control is possible with needle valves
        - http://pergatory.mit.edu/2.007/handouts/actuator/piston/piston.html
        - *Remember to use clevis' at ends for couplings!*





Pneumatic Systems: *Energy Storage*

A pneumatic piston can be used as a spring, where the amount of work W done in compressing the cylinder of diameter D and initial length L by an amount x can be easily determined. If it is first assumed that the temperature of the gas does not change due to slow compression, the process is *isothermal*, then according to the ideal gas law, the product of the initial pressure and volume equals the product of the final pressure and volume:

$$P_i V_i = P_f V_f \;\Rightarrow\; P_{x=0}\frac{\pi D^2 L}{4} = P_x \frac{\pi D^2 (L-x)}{4} \;\Rightarrow P_x = \frac{P_{x=0}L}{L-x}$$

$$F_x = A\left(P_x - P_{x=o}\right) = \frac{\pi P_{x=0}D^2}{4}\left(\frac{x}{L-x}\right)$$

$$W = \int_0^x F_x dx = \frac{\pi P_{x=0}D^2}{4}\left(-x - L\ln(-L+x)\right)\Big|_0^x = \frac{\pi P_{x=0}D^2}{4}\left(L\ln\left(\frac{L}{L-x}\right) - x\right)$$

What about when the cylinder is rapidly compressed, so there is effectively no heat transfer to the cylinder walls? Then γ is typically about 1.7 and:

$$P_i V_i^\gamma = P_f V_f^\gamma \Rightarrow P_{x=0}\left(\frac{\pi D^2 L}{4}\right)^\gamma = P_x\left(\frac{\pi D^2 (L-x)}{4}\right)^\gamma \Rightarrow P_x = P_{x=0}\left(\frac{L}{L-x}\right)^\gamma$$

$$F_x = A\left(P_x - P_{x=o}\right) = \frac{\pi P_{x=0}D^2}{4}\left(\left(\frac{L}{L-x}\right)^\gamma - 1\right)$$

$$W = \int_0^x F_x dx = \frac{\pi P_{x=0}D^2}{4}\left(-x + \left(\frac{L}{L-x}\right)^\gamma\left(\frac{L}{\gamma-1} + \frac{x}{1-\gamma}\right)\right)\Bigg|_0^x$$

$$= \frac{\pi P_{x=0}D^2}{4}\left(-x + \left(\frac{L}{L-x}\right)^\gamma\left(\frac{L}{\gamma-1} + \frac{x}{1-\gamma}\right) - \frac{L}{\gamma-1}\right)$$

How much energy E is stored in a gas cylinder used in a 1 atm. *(P_a)* environment when it is pressurized to an absolute pressure P_c, where the temperature in the cylinder is kept constant. Assume it has a diameter D_c and length L_c (or volume V)? Imagine moving from an extended to a contracted (pressurized) state:

$$L_i = L_e = L_c\frac{P_c}{P_e} \qquad x = L_c\left(\frac{P_c}{P_e} - 1\right)$$

$$E = \frac{\pi D_c^2 L_c}{4}\left[\frac{P_c}{P_e}\left(\ln\left(\frac{P_c}{P_e}\right) - 1\right) + 1\right]P_e \qquad \Rightarrow V\left[\frac{P_c}{P_e}\left(\ln\left(\frac{P_c}{P_e}\right) - 1\right) + 1\right]P_e$$

The above are used in the spreadsheets *pneumatic_cylinder.xls* and *pneumatic_pressurized_cylinder.xls.* The latter shows that a 2 liter soda bottle pressurized to 6 atm. of pressure stores energy equivalent to 288 constant force springs typically provided in robot contest kits (10N x 400 mm)! This is an incredible amount of energy for a small machine, and if that much energy were suddenly released by rupture of the soda bottle, severe bodily damage could occur. 3 atmospheres of pressure in a 50 ml syringe is the equivalent of 2 constant force springs. Note the amount of energy absorbed by a cylinder that is used as a shock absorber in mountain bike suspensions. Were the inventors of suspensions lucky, or did they do their homework?

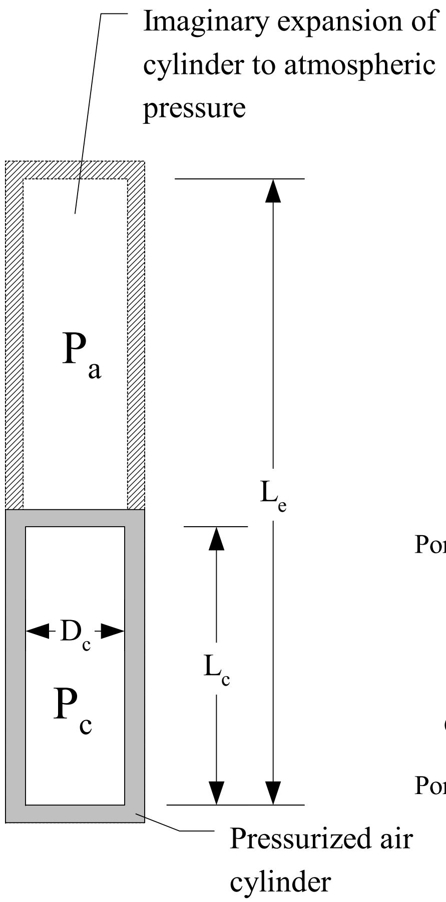
Consider the energy stored in a 50 ml plastic syringe that is compressed to 10 ml. What is the final pressure and how much energy is stored? How does this compare to filling the syringe up with 3 at of pressure and just using the syringe as a storage cylinder? If the gas is to be released to power a piston, why not just use the piston itself? Can the syringe port be blocked (releasably, why?) and then the piston compressed and held compressed with a trigger? If the cylinder is initially compressed (adiabatically) the force will be high, but then as the gas cools it will drop. How would this affect your plans? Experiment with the spreadsheets!

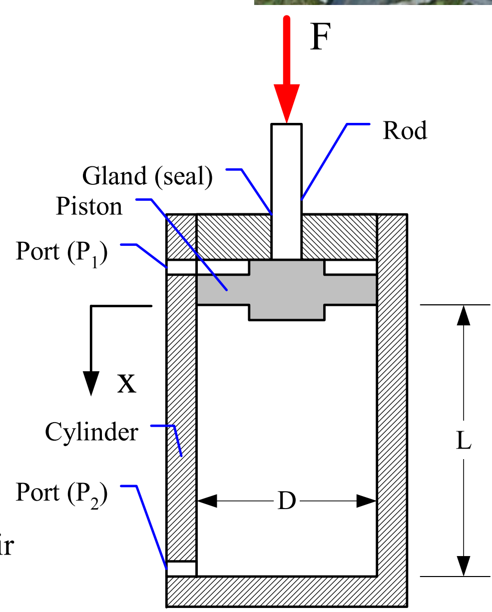
If your contest does not provide an umbilical or pressurized gas cylinders, can you use simple plastic syringes? What is their failure mode and is it safe? How would you test them to make sure they will not explode at the intended pressure? How can you initially preload (pressurize) them and then get the energy back? If you are allowed a pressurized gas cylinder, how can you use the above equations, or modify the spreadsheet to predict how much useful work you can get out of the devices that you will be actuating? Will you have enough pressure and hence force for many cycles?

Pneumatic Systems: *Energy Storage*

- How much energy is stored in a pressurized air cylinder?
- How much energy can a pneumatic piston absorb when it is used as a shock absorber or as a spring?
  - ***NEVER use PVC pipe as a pneumatic cylinder or energy storage device (see page E-15)***

Pneumatic_pressurized_cylinder.xls			
To determine energy storage in a pressurized cylinder			
By Alex Slocum & Roger Cortesi 1/20/04, last modified 2/16/04 by Alex Slocum			
Enters numbers in **BOLD,** Results in RED			
Cylinder as energy storage device			
Diameter, Dc (mm)	**25**		
Length, Lc (mm)	**101.9**		
Atmospheric pressure, Pa (atm, N/mm^2. psi)	**1**	100000	14.7
Cylinder pressure, Pc (atm, N/mm^2, psi)	**3.0**	300000	44.1
Volume, Vc (mm^3)	**50000**		
Imaginary length to expand to obtain 1 atm pressure, Le (mm)	306		
Energy stored, Ec (Joules=Nm)	6		
Compare to a constant force spring:			
Force, Fspring (N)	**10**		
Spring length, Lspring (m)	**0.40**		
Energy stored, Espring (Joules=Nm)	**4**		
Equivelent number of constant F springs	2		
Height to which a 100 kg professor can be raised (m)	0.00661		





Pneumatic_cylinder.xls			
To determine forces and energy in a pneumatic cylinder (piston)			
By Alex Slocum & Roger Cortesi, last modified 1/20/2004 by Alex Slocum			
Enters numbers in **BOLD,** Results in RED			
Cylinder diameter, d (mm)	**20**		
Rod diamter, dr (mm)	**5**		
Piston stroke (maximum), L (mm)	**50**		
Total volume, V (mm^3)	15708		
Differential area, Ad (mm^2)	295		
Pneumatic cylinder as an actuator			
Piston-side presure, P (atm, N/m^2, psi)	**6**	600000	88.2
Rod-side pressure, Pr (atm, N/m^2, psi)	**1**	100000	14.7
Net pressure force on piston, Fp (N)	143		
Pneumatic cylinder as a compression spring: ISOTHERMAL (constant temperature)			
Initial cylinder pressure, P_1 (atm, N/m^2, psi)	**1**	100000	14.7
Displacement (amount piston compresses gas), x (mm)	**20**	**15**	**10**
Pressure (atm)	1.7	1.4	1.3
Force (N)	21	13	8
Energy absorbed (Joules=Nm)	0.174	0.089	0.036
Pneumatic cylinder as a compression spring: ADIABATIC (NO HEAT TRANSFER)			
Gas (air) properties			
Temperature of the gas at start, T_1a (^{o}C)	**20**		
R, Rgas (J/(kg-K))	**2078**		
cv (J/(kg-K))	**3153**		
γ, gamma	1.65905		
Initial cylinder pressure, P_1a (atm, N/m^2, psi)	**1**	100000	14.7
Displacement (amount piston compresses gas), xa (mm)	**20**	**15**	**10**
Pressure (atm)	2.3	1.8	1.4
Force (N)	42	25	14
Energy absorbed (Joules=Nm)	0.326	0.160	0.063
Temperature of the compressed gas (^{o}C)	28.0	25.3	23.2

Pneumatic Systems: *Implementation*

Pneumatic cylinders can be used as active elements, where a valve is controlled to control the flow of air into the cylinders. They can also be used as passive elements where they have been compressed so they act like springs, and their energy can be released by a trigger. In either case, there are three primary practical issues associated with the use of pneumatic systems: *Seals*, *mounts*, and *hoses*. *Seals* keep the pressure in and the dirt out. *Mounts* must make sure that the actuator is not overconstrained, or the generated actuation force can end up destroying the actuator by creating lateral forces and moments. *Hoses* deliver the air, but can also cause entanglement.

There are many types of seals, and most use the *principle of self-help* to seal in pressure: As the pressure increases, it deforms the seal, causing it to push harder on the piston and cylinder wall, or to squeeze the rod tighter. This also means that seal friction is a major contributor to system inefficiency. Lubrication is not always a good thing though: Seals are among the most critical of pneumatic (and hydraulic) system components, because they must keep dirt out, despite the fact that the motion of the rod tries to drag dirt in. Any dirt that gets by the seal can scratch the rod or cut the seal and cause a leak. The ideal lubricant is dry (e.g., Teflon). With proper low-friction seals available from a catalog, it is not too difficult to design and manufacture a specialized cylinder, although there are so many manufacturers, it is not often that a custom design is needed.

O-rings are one of the most common types of seal used in pneumatic systems. As described on Apple Rubber's website[1], "An O-ring is a doughnut-shaped object, or torus. The opposite sides of an O-ring are squeezed between the walls of the cavity or "gland" into which the O-ring is installed. The resulting zero clearance within the gland provides an effective seal, blocking the flow of liquids or gases through the gland's internal passage." An O-ring must have some initial deformation in order for it to seal. Once pressure is applied, it deforms further, and the principle of self-help takes over. An O-ring should never pass over a hole in a structure, such as a port hole, preferably not during installation and certainly never during use; as this could cut it.

1. See www.applerubber.com. This website and those of other seal manufacturers provide extensive design detail because they want you to use their products properly, so you will be happy with them!

O-rings' hardness is given by their *durometer* (*Shore A* is one test method) which is a measure of the elastomer's hardness. Softer materials are less than about 70, harder materials are greater than about 90. Higher durometers are used at higher pressures. Softer durometers are more conformal.

Pneumatic cylinders are usually purchased[2], and care must be taken to properly couple the ends to mounting points so the cylinder is not overconstrained (see page 5-29). If overconstrained, efficiency will be decreased as some of the actuation force will go to deforming the actuator. Also, just as electrical lines require strain relief and need to be anchored in place to prevent them from being torn off, so do pneumatic lines. Any well-made machine will have all cables and hoses neatly routed and anchored with generous strain reliefs. If you see a rat's nest of cables and wires, stay away!

Review the FRs for any pneumatic systems, and carefully consider actuator performance in your power budget. Safety first!

2. For the MIT 2001 2.007 contest *Tiltilator*, Will Delhagen & Alex Jacobs made their own large pistons which drew power from the optional-use umbilical. The goal of the contest was to get the beam to tilt down on your side. Different strategies included extending weights, lowering a grabbing claw down to the carpet to pull down the beam, and moving to the opponent's side to jack the beam up. Other students sought to knock off the opponent and then patrol the beam and race to their end at the last minute. Several students decided upon the concept of dropping to the floor and driving over and under the opponent's beam. Some designed screw-jacks which were powerful, but slow. Will & Alex each noted the fact that a pneumatic piston can travel relatively fast when there is no load on it. They were planning to roll their aluminum sheet into cylinders. The piston seal would be formed by placing a piece of rubber between two plates, thus forming a kind or a lip seal or "packing" seal. Rolling the aluminum sheet into a cylinder and then making a joint that would not leak, and that would allow the piston seal to seal proved extremely difficult. A number of students had been asking if they could swap equivalent amounts of material for different shapes. The teaching staff decided to allow this as an "experiment" (it became a logistical nightmare), and so Will & Alex swapped their sheet for equivalent weight thin-walled tube.

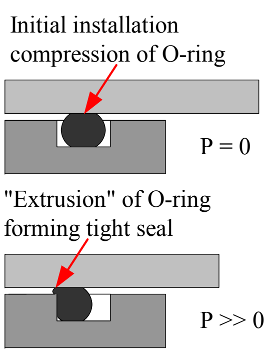
Will and Alex built their pistons by welding flat end caps onto the thin walled cylinders, which the welder said "that should hold it"; however, because of the size of the pistons, about 75 mm in diameter, they now had a pressure vessel. They had done calculations for the hoop stresses in the cylinders, but they had not calculated the stresses in the flat plates that covered the ends (see pages 8-23 and 8-25). Using Roark's *Formulas for Stresses and Strains*, Prof. Slocum showed them that the material could yield! They were given permission to proceed if they would make the design far more conservative and read the ASME pressure vessel code.

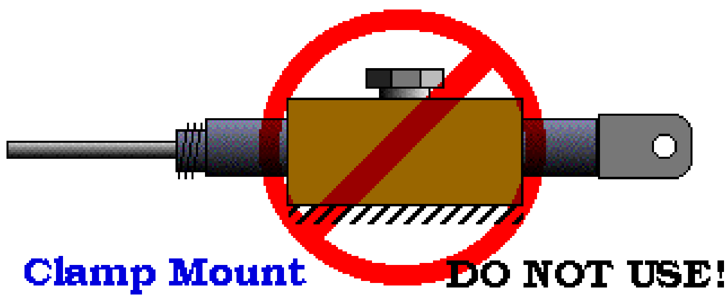
Ultimately, Will and Alex faced each other in the final round, and they asked Prof. Slocum if there could be a tie. The answer was "no". So during preparation for the final round, they went to the preparation area and quickly installed hard-stops so that each piston would extend the same amount. They positioned themselves under the beam and lifted up the entire 400 pound contest structure! A level showed the beam was exactly level. They were declared co-winners.

Pneumatic Systems: *Implementation*

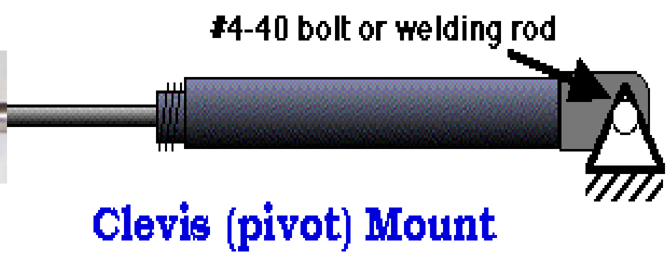
(OK, this picture shows hydraulic pistons, but what ideas does it help you dig up?)

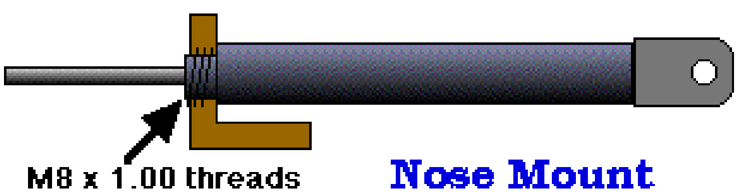
- It is extremely important to avoid over constraint when mounting a cylinder and connecting the rod to the load
  - Clevises provide a means to prevent over constraint
  - DO NOT squeeze the cylinder body or use a set screw!
  - This can change the bore and prevent motion of the piston!
- Seals are a source of friction, lubricate to reduce friction: some lubricants attack seals, and lubes attract dirt, so use a dry lubricant (e.g., Elmer's Slide All™ a Teflon powder lubricant)

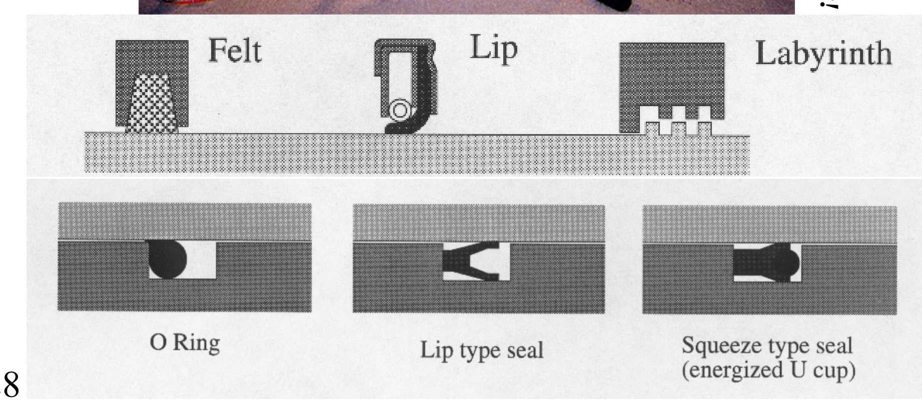




Will Delhagen & Alex Jacobs in MIT's 2001 2.007 contest *Tiltilator!*







Power Budgets

Your machine's power supply must have the instantaneous power to perform required motions, as well as the total energy required for all required motions. More than one brilliant machine has limped along at a fraction of the intended speed because the student did not do a preliminary *power budget*, which would have identified problem areas. In fact, because the strategy phase should involve simple stick-figures and diagrams with estimates of forces and speeds (see page 2-11), it is simple to construct a preliminary power budget, such as shown in *Power_budget_estimate.xls*. Once the best strategy is selected and concepts are developed, a more detailed power budget should be created. *Power_budget_estimate.xls* can again be used, but it is expected that there will be greater detail and certainty. Again, the power budget can help in the selection of the best idea.

Once the best concept is selected and the detailed design commences, a detailed final power budget can be created. The detailed power budget can for example use results from applications of *Gearmotor_move.xls* for each motor, or an entirely new custom spreadsheet can be created.

One of the biggest mistakes made in developing power budgets is the mix-up of units. This is not without historical precedent! Recall the scenes from the movie Apollo 13, where the engineers on the ground are told that they have to get the power down to less than 13 amps... Such mistakes with units can cause serious problems. Recall the Mars Voyager that tried to land below the surface of mars because of a mix up with kilometers and miles. Do not let this happen to you:

- Power: force (Newtons) x velocity (meters/second) = Watts.
- Energy: Power (Watts) x time (seconds) = Joules (N-m) or Force (Newtons) x distance (meters) = N-m (Joules).
- For rotary systems, replace force with torque (N-m) and linear velocity with angular velocity (radians per second).
- Trouble often occurs when rpm is used instead of radians per second (rad/s = rpm*2π/60). Radians are used for all power and energy calculations!

Another issue that often leads to errors is overestimating the efficiency. For *Power_budget_estimate.xls*, the efficiency is the product of the transmission system efficiency and the electrical efficiency of the motor. Recall that the electrical efficiency of the motor is typically at most 50%, which means if the required mechanical power from the motor is 2 Watts, then the motor will draw at least 4 Watts of electrical power from the batteries. Often the motor is not operating at its peak electrical efficiency, such as when it is operating at its peak mechanical efficiency, the electrical efficiency may be as low as 35%. Combined with a 4 stage planetary transmission system efficiency of 95% per stage, the total system efficiency may be 29%. The batteries have to provide three times the power that the simple mechanical move requires. In fact, for preliminary design purposes, it is wise to assume that 3-5 times the pure mechanical move power is required of the batteries.

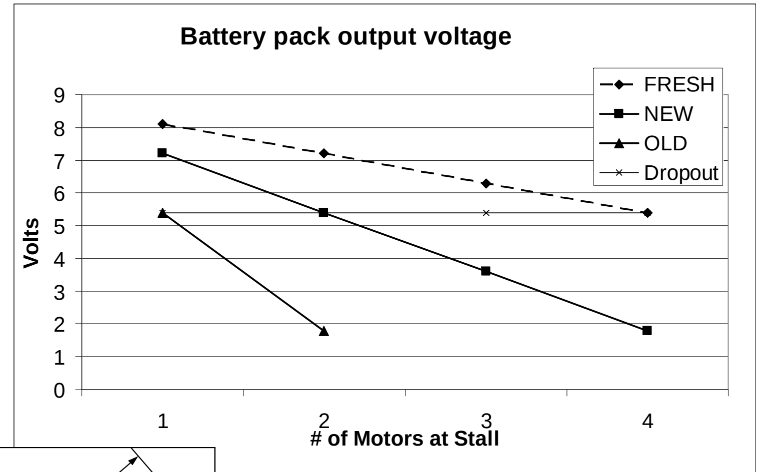
The batteries also have internal resistance which must be considered in terms of the overall power budget. Recall the spreadsheet batteries.xls, which is incorporated into *Power_budget_estimate.xls* so an estimate of the battery power dissipation can be made for each move. Note that the power dissipated in the batteries internal resistance heavily depends on the total system electrical resistance, which depends on the number of active motors. The number of active motors can be entered into the spreadsheet. Since the power dissipated in the batteries is $I^2R_{\text{battery internal resistance}}$, there is a huge power dissipation difference between disposable alkaline and rechargeable NiCad batteries! The former in general are economical for circuits with high resistance (low current). Note that some alkaline batteries are available for higher current applications, but their internal resistance is still many times higher than that of a NiCad battery. Where high currents are drawn, NiCad batteries are far more efficient. What about other types of batteries?

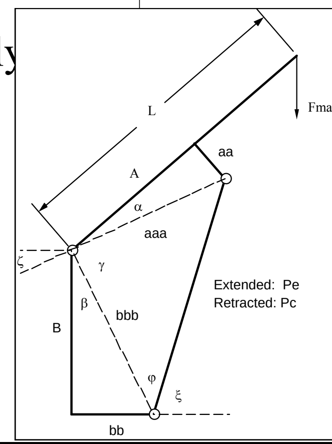
The method used in the spreadsheet provides a good estimate of system power and energy; however, for a mission critical application, a detailed system dynamic model would have to be created that includes all forms of dissipative losses and functions performed.

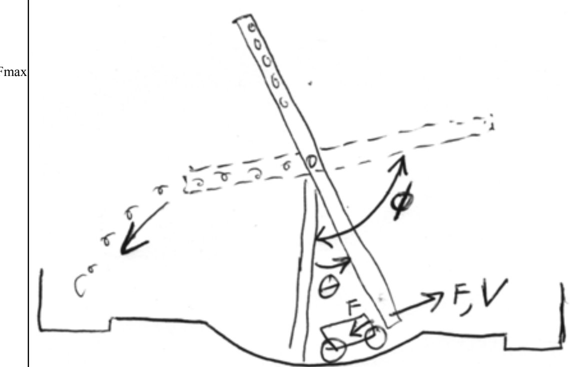
Create a power budget for your machine idea. Ideally you will be reading this page having jumped here from Topic 1, and will create a power budget for your strategies and then your concepts on the way to selecting the best idea to detail. In addition to the electrical system power, remember to consider mechanical system power that is to be supplied by springs or compressed air! Modify *Power_budget_estimate.xls* as needed.

Power Budgets

- Each machine move requires power (force x velocity) and energy (force x distance)
  - Do you have the power?
    - Experiment with *Power_budget_estimate.xls* and *Power_to_Move.xls*
  - Do you have enough energy?
  - Which motions can you accomplish simultaneously
- How can you tell if you have enough resources?
  - Create a *Power Budget*
- Beware of internal battery resistance
  - NiCad vs Alkaline…
  - Experiment with *Power_budget_estimate.xls*







Power_budget_estimate.xls										
Power & energy budget for individual moves, total (S) for simultaneous moves, and cumulative										
Last modified 9/01/03 by Alex Slocum										
Enters numbers in **BOLD**, Results in RED						Power (Watts)			Energy (N-m)	
Axis	Move #	Force (N)	Velocity (m/s)	Distance (m)	Efficiency, net system	Move	Battery dissipation	Σ power for move #	Energy for move	Σ Energy
Drive to pucks	**1**	**3**	**0.2**	**1**	**29%**	**2.10**	**8.30**		**52.0**	**52.0**
Lower arm	**1**	**0.5**	**0.5**	**0.04**	**29%**	**0.88**	**8.30**	**11.28**	**0.7**	**52.8**
Scoop	**2**	**3**	**0.2**	**0.02**	**29%**	**2.10**	**3.00**	**5.10**	**0.5**	**53.3**
Raise arm	**3**	**3**	**0.2**	**0.05**	**29%**	**2.10**	**3.00**	**5.10**	**1.3**	**54.5**
Drive to goal	**4**	**2**	**0.2**	**0.5**	**29%**	**1.40**	**3.00**	**4.40**	**11.0**	**65.6**
Dump pucks	**5**	**0.1**	**0.5**	**0.05**	**29%**	**0.18**	**3.00**	**3.18**	**0.3**	**65.9**

Case Study: *System to Measure Gearmotor Performance*[1]

How do you get the torque-speed curves, such as those shown for gearmotors, when they are not provided by the manufacturer? How can you verify such curves given by the manufacturer? How can you study the effect of wear-in which usually results in lower brush resistance and thus higher torque potential? The answer is to create a low-friction test system.

A flexible coupling or low-cost double universal joint (see page 5-29) is used to connect the gearmotor's output shaft with the load shaft. This ensures that the gearmotor's output shaft experiences no moment loads, as should be the case for when the gearmotor is used in practice. To illustrate trade-offs in the design process, the design is shown with a either sliding contact bushing as the support bearing for the loaded shaft, or ball bearings.

The simplest design uses sliding contact material, such as Teflon, as a bushing (bearing). Unfortunately, the friction from steel-on-Teflon can cause significant friction torque which can be a primary source of error in the measurements. The friction torque can be estimated by determining the radial forces on the bearings and then determining the friction torque using the coefficient of friction and the shaft radius:

$$\sum M = 0 = F_1 L_2 + F(L_1 + L_2) \qquad \sum F = 0 = F_1 - F_2 - F$$

$$F_1 = \frac{F(L_1 + L_2)}{L_2} \qquad F_2 = \frac{FL_1}{L_2}$$

$$\Gamma_F = \mu \frac{D}{2}\frac{F}{L_2}\left(2L_1 + L_2\right) = \frac{\mu DF\left(2L_1 + L_2\right)}{2L_2}$$

$$D = 6mm, L_1 = 22mm, L_2 = 21mm, \mu_{\text{bushing}} = 0.05, \mu_{\text{ball bearing}} = 0.01, F = 40N$$

$$\Rightarrow \Gamma_{\text{Bushing friction torque}} = 18.6Nmm, \Gamma_{\text{Ball bearing friction torque}} = 3.7Nmm$$

What sensitive parameters do the equations illustrate? What design opportunity presents itself with the system shown? The plot shows how the friction torque decreases with increasing L_2/L_1 ratio, and at a ratio of 3-5 there is little more to be gained. Saint-Venant rules! (see page 3-5 for a refresher if needed). Although $L_2 = 3.5D_{shaft}$, and thus seemingly adhering to Saint-Venant's principle, why was this system made with $L_2/L_1 = 1$ which does not? The plot shows that for a sliding contact bearing the friction torque is unacceptable at $L_2/L_1 = 1$. However, the calculations show that with ball bearings, the friction torque would be only about 1% of the total torque. Since the parts where already available, the acceptable decision was to proceed with these proportions. This illustrates the point that guidelines should be conservative, but not limiting for those who know the detailed physics behind them, and who are willing to investigate the details!

What other experiments can be conducted with such a test stand? One could test the selection of the optimal transmission ratio and the motor and load power ratings. Life tests can also be conducted, including the resistance of the brushes as they wear-in, and the effect of various levels of stall torque and stall time on motor performance. If a motor were being selected for a critical application, the design could proceed, leaving space for the various candidate motors, while extensive tests were completed on this type of test stand. In addition, battery and control system performance could also be evaluated. One could even evaluate different types of bearings and couplings, and compare system efficiency to a baseline "perfect" system.

In the end, when a system is run and data gathered for analysis, all data is real: It is the understanding and interpretation of the data that sometimes gets confused. The key to being able to interpreting the data is a sound analytical and physical understanding of the physics of the experiment *and* of the experimental apparatus.

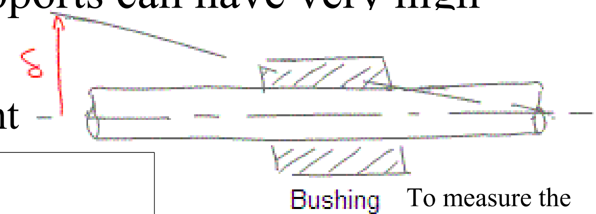
Do you have enough gearmotor, battery, and control system data to proceed with your design without too much risk? What tests would you like to run on your motors? What is the appropriate level of testing needed in order for you to proceed with a design to meet your scheduled milestones? At what point do you accept a lower performance machine, but one that is delivered on-time and with less risk, but is "guaranteed" to work reasonably well? Have you ever bought a state-of-the-art super high performance product only to have it continually being repaired?

---

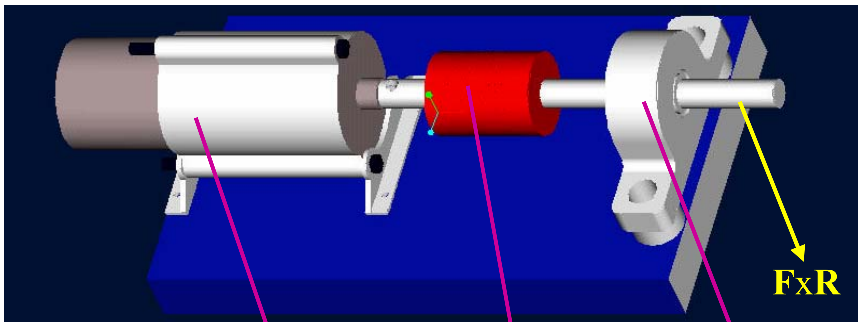
1. Many thanks to my graduate student and dear friend Fadi Abu-Ibrahim for developing this experiment!

Case Study: *System to Measure Gearmotor Performance*

- Gearmotor suppliers do not always provide you with the data you need
- A test system that uses proper couplings and ball-bearing supports can have very high efficiency and thus yield good data
  - Motor efficiency = (mechanical power)/(motor voltage x current

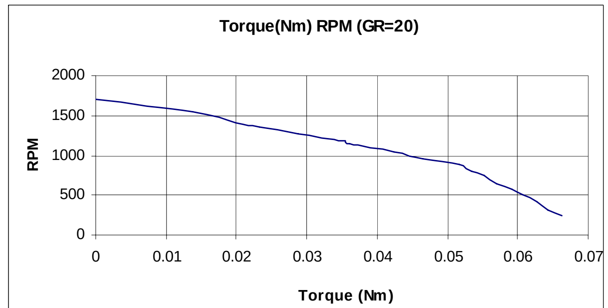


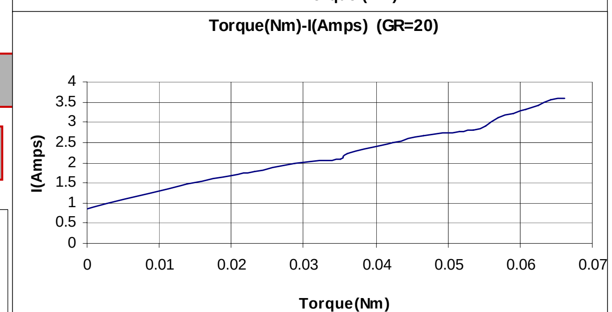
To measure the static friction of a sliding contact bushing, tilt the bushing holding a shaft until the shaft starts to slide, and μ = the tangent of the angle

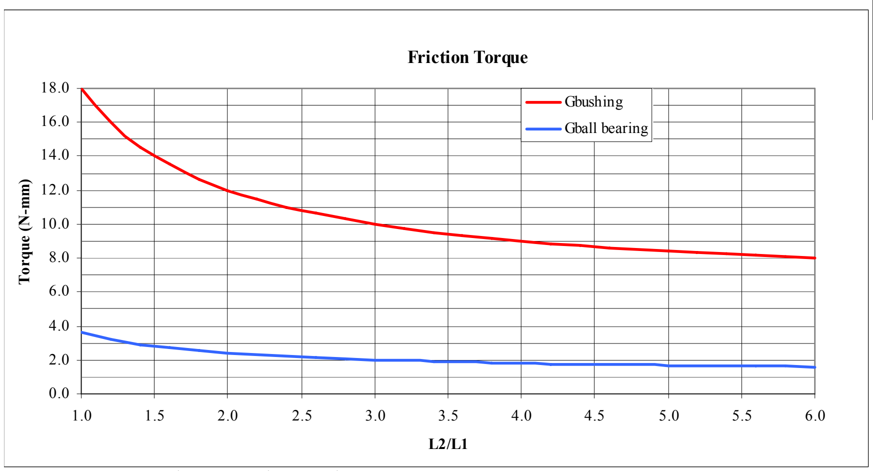


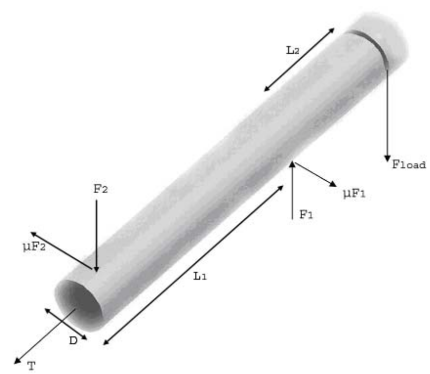
Design One Gearmotor → Flexible Coupling → 2 Ball bearings

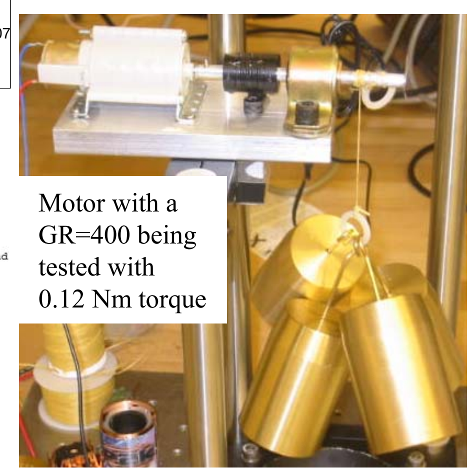
Design Two Gearmotor → Flexible Coupling → Teflon bushing











Motor with a GR=400 being tested with 0.12 Nm torque

Topic 7 Study Questions

Which suggested answers are correct (there may be more than one, or none)? Can you suggest additional and/or better answers?

1. A transmission is used to convert power from one form into another
   True
   False
2. There is an "optimal" transmission ratio that maximizes system efficiency, and it is on the order of the square root of the ratio of the load inertia to the motor rotor inertia:
   True
   False
3. Ohm's law: Voltage (electromotive force) in a circuit = current x resistance (E = IR)
   True
   False
4. The magnetomotive force F_m (MMF) in a magnetic circuit is proportional to the magnetic flux Φ (flux) and the reluctance R (Fm= ΦR)
   True
   False
5. Kirchoff's current law: The sum of all currents (fluxes) flowing into a node is zero
   True
   False
6. Kirchoff's voltage law: The sum of all voltage drops (MMF) in a closed loop equals zero
   True
   False
7. Faraday's law of electromagnetic induction: Coils of wire and magnets interact to create electric and magnetic fields
   True
   False
8. A force F is required to move a conductor of length L carrying a current i through a magnetic field of strength B (F = BLi)
   True
   False
9. A magnet that moves in a coil causes a potential difference at the terminals of the coil
   True
   False
10. Ampere's law: An "electromotive force", such as created by current passing through a coil of wire, forces a magnetic field through a magnetic circuit
    True
    False
11. The product of the magnetic field intensity H and length in a circuit equals the magnetomotive force:
    True
    False
12. Gauss's law for magnetism: Magnetic fields have North & South poles between which the field flows:
    True
    False
13. The total magnetic flux (Φ=B*Area) across a closed boundary is zero (there are no magnetic monopoles):
    True
    False
14. Magnetic flux Φ flows in a circuit, just like electricity:
    True
    False
15. The permeability μ = B/H is the ability of a magnetic field to permeate (flow) through a material
    True
    False
16. $\mu_0 = 4\pi \times 10^{-7}$ (tesla-meter/ampere) = permeability of free space and non-magnetic materials

True
False
17. Reluctance R is a measure of the resistance to flow of magnetic flux through a particular piece of material:
True
False
18. Permanent magnets provide magnetic flux which flows from their North pole to their South pole
True
False
19. In a circuit, permanent magnets act as a “voltage” source, MMF = Fm=BL/mo
True
False
20. The “current” (flux density), B=Φ/A, associated with the “voltage” depends on the rest of the circuit
True
False
21. If too much “current” is drawn, permanent magnets will demagnetize
True
False
22. Solenoids use a coil to generate a magnetic field that then attracts an iron plunger
True
False
23. Solenoids have limited pull force and stroke and are often used to open or close devices or as trigger actuators
True
False
24. Lorentz forces are created either from differential charges between objects (from Coulomb’s law) or by a coil in a permanent magnet field
True
False
25. With a uniform magnetic field over a wide region, a reasonably constant force can be generated in a current carrying wire that moves within the field
True
False
26. Electric motors convert voltage into speed and current into torque
True
False:
27. An estimate of rotary motor torque can be made from the product of motor rotor surface area, rotor radius and electromagnetic shear stress (on the order of 0.2 atm for a simple dc motor):
True
False
28. Peak power from a voltage controlled DC motor occurs at one-half the maximum speed:
True
False
29. Output shafts and their support bearings will generally support small robot machine radial loads if they are not too far overhung:
True
False
30. A leading cause of motor (and system) failure is too large radial loads applied to a motor shaft:
True
False
31. HEAT (thermal overload) is one of the greatest causes of motor damage
True
False
32. Since pumps and power supplies are placed external to a machine, they can be selected based on minimum purchase price:

True
False

33. In today's modern digital age, electromagnetic interference (EMI) is no longer a problem:
    True
    False
34. Linear electric motors are perfect force sources and since they only have one moving part, separated from the other by a simple air gap, they are not subject to most of the problems associated with ball screws:
    True
    False
35. Heat from linear electric motor coils can be a major problem, and thus it must be carefully managed by design:
    True
    False
36. The order of preference for managing the heat from linear electric motor coils is passive (heat sink), active (forced air cooling around and through the coils), and active (coolant circulated through the coils):
    True
    False
37. Open-face iron-core linear electric motors generally provide the lowest cost per Newton of actuation force available from linear electric motors:
    True
    False
38. Open-face iron-core linear electric motors are easy to mount, but the attractive force between the coils and the magnets is on the order of 5x the actuation force they provide, and hence the load they place on linear bearings can be a problem:
    True
    False
39. If the motor coils and magnets are positioned properly in a system, they can be used to provide even preload to bearings arranged:
    True
    False
40. The principle of self-help can be applied to cancel the attractive force between open-face iron-core linear electric motor components by using two motors placed back-to-back
    True
    False
41. Ironless core linear electric motors use two rows of opposing magnets with an ironless core forcer riding between them to generate actuation force without causing significant lateral forces:
    True
    False
42. All springs provide constant power:
    True
    False
43. Coil springs generally provide force proportional to their displacement:
    True
    False
44. Pneumatic (and hydraulic) cylinders let energy be piped in from a distant source:
    True
    False
45. Fluid pressure (pneumatic or hydraulic) in a cylinder acts on a movable surface (the piston) which generate a force that is then transmitted through the rod
    True
    False
46. NEVER use PVC pipe as a pneumatic cylinder or energy storage device
    True
    False
47. It is extremely important to avoid over constraint when mounting a cylinder and connecting the rod to the load or else efficiency will be lost and the system may jamb:

True
False
48. Clevises provide a means to prevent over constraint:
True
False
49. DO NOT squeeze a pneumatic cylinder body (where the piston travels) or use a set screw to anchor it:
True
False
50. Seals are a source of friction, lubricate to reduce friction: some lubricants attack seals, and lubes attract dirt, so use a dry lubricant
True
False
51. Each machine move requires power (force x velocity) and energy (force x distance) and thus requires a power budget to ensure that the power sources and actuators can enable the machine to meet its performance requirements:
True
False
52. Underestimating or not accounting for friction is a primary source of error in power budgets:
True
False
53. Position sensors should usually be placed near the center of stiffness of a moving component:
True
False
54. As long as they are placed in a cable carrier, sensor and motor power cables do not have to be separated from each other:
True
False
55. Sensor mounting systems do not have to be concerned with stiffness because they bear no loads:
True
False
56. Sensor mounting systems should be an integral part of the machine's design and not designed after the rest of the machine has been detailed:
True
False
57. Wiring, hoses, and cable carriers can all be added by manufacturing personnel after a design is complete because trying to plan where to route lines is never accurate and assembly people know best:
True
False
58. Linear encoders are often a good displacement sensor to use in machines used in a non-temperature controlled environment because they can be selected to expand with the structure thereby helping to ensure that the component manufactured on the machine will "shrink" back to a standard reference length when it is brought to standard temperature:
True
False
59. Standard temperature is 20 ^{o}C (68 ^{o}F):
True
False
60. Changes in barometric pressure affect readings to the first order from:
Laser interferometers
Optical encoders
Magnetic encoders
61. Laser interferometers always provide the best possible measuring means because they use the standard of the wavelength of light as their reference:
True
False